

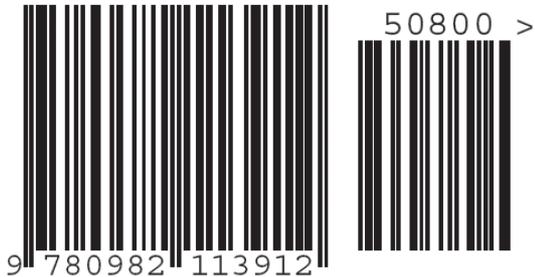
# Handbook of Land and Tree Measurements



Handbook of Land and Tree Measurements

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Harley Langdale Jr. Center  
for Forest Business  
UNIVERSITY OF GEORGIA

# HANDBOOK OF LAND AND TREE MEASUREMENTS

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**Harley Langdale Jr. Center  
for Forest Business  
UNIVERSITY OF GEORGIA**

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This book is dedicated to all of the former University of Georgia  
Warnell School of Forestry and Natural Resources students who  
endured *Field Orientation, Measurements, and Sampling*

## 1.0 INTRODUCTION TO LAND AND TREE MEASUREMENTS

The need for forestry students to understand how land and trees should be measured or sampled should be obvious. However, students in fisheries, wildlife, recreation, and soil and water management also need these skills. A few examples of the need for other majors to understand these techniques include the following:

1. There is a strong relationship between landscape attractiveness and the presence of large trees, particularly when the trees are 15 inches or greater in diameter (Jaakko Pöyry Consulting, Inc. 1993, Rudis et al. 1988).
2. Tree diameter and tree distance from a stream were data collected for a 2004 study on wadeable streams in Mississippi (Alford and Jackson 2006), and trees within the riparian area are generally considered future large woody debris for the stream system (Oregon Department of Forestry 1995).
3. Canopy structure around streams influences solar radiation reaching the stream, as well as primary and secondary production of fish food within the streams (Wipfli 2005).
4. Bass catch per unit effort has been shown to be negatively related to the diameter of conifer trees along streams, and positively related to the diameter of hardwood trees (Alford et al. 2005).
5. Riparian habitat condition is expressed in part by the number of large conifers near the stream system (Oregon Department of Fish and Wildlife 1999).
6. Some hatchery management plans describe the basin within which the hatchery is situated, and use metrics such as amount of land in various successional stages, as categorized by trees in different diameter classes (U.S. Fish and Wildlife Service 2005).
7. Basal area is a measure used in relation to recommended harvests in timber stands with wildlife objectives (Missouri Conservation Department 2008, Gano 1991), and for evaluating the quality of red-cockaded woodpecker habitat.

8. Basal area, trees per acre, snags per acre, crown closure, and other tree-related metrics are used in a large number of wildlife habitat suitability (HSI) models.

The main point is that students in each major need to know (a) how to measure land and trees, and (2) how to sample tree characteristics. We need these skills in order to arrive at reasonable estimates of forest resources to address issues such as those noted above.

At the very least, you will eventually be working with other people who are performing tasks related to land and tree measurements. You need to be able to communicate with them effectively, and one way to do this is to know how they measure land and trees. Fisheries biologists, wildlife managers, soils and water specialists, recreation managers and foresters all work together to manage natural resources.

The topics covered in this book include the following:

1. An overview of land survey systems. This provides you with a sense for how land was initially surveyed, and how you can obtain survey information from county courthouses. This information should be valuable in your personal life as well, if someday you plan to purchase or sell real property.
2. The common methods we use in natural resource management for measuring land. This includes compass and pacing, and the processes used to convert field measurements to maps. An exposure to global positioning systems (GPS) is provided, along with overviews of mission planning, differential correction, and accuracy assessments.
3. The common methods for sampling trees in natural resource management, including the use of fixed area plot, fixed area strip, and variable radius point (prism) sampling. A number of examples are provided to show how these field measurements

can be converted to information that is commonly used in natural resource management planning.

4. The basic statistical methods for examining the variation in field measurements, and for developing bounds on the estimates of natural resource characteristics at the tree- and stand-levels.

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## 2.0 LAND SURVEY SYSTEMS

From the beginning of settlement in North America, people either claimed or were granted land. This land had to be legally described to establish ownership, facilitate transfer of ownership, and provide a basis for taxation. Several systems were used in the original 13 British colonies and in areas governed originally by the Spanish or French. The Spanish *vara* system is often encountered in the Gulf coastal states, California, Arizona, and New Mexico. French systems are often found in Gulf coastal states, the Lake States, and parts of New England. Much of the remainder of the United States was described using the Public Land Survey System. Therefore, across the United States a variety of systems may be encountered, depending on the time of first settlement and the country claiming the land.

Three systems were used to grant land to settlers in Georgia:

- Royal Headright
- State Headright
- State land lotteries

Approximately 1/3 of Georgia was granted to settlers using the Royal and State Headright systems. The original surveys were described using *metes and bounds*. The remainder of the state was granted to settlers using the State Lottery system. This system used square tracts to allocate land to settlers.

After completing this Chapter, you will be able to:

1. Understand the history of land surveys in Georgia.
2. Understand the details associated with the Public Land Survey System.
3. Understand the various terms and records associated with county courthouse records.

## 2.1 METES AND BOUNDS

All of the original surveys in Georgia were described by *metes and bounds*. It is a system of describing real estate based on English Common Law. This system involves describing the boundaries of a property by physical land features such as streams, trees, rocks, etc. While the system today is inadequate, due to the subjective and transient nature of physical features, many of the original surveys were described in this manner. As time progressed, physical features were replaced by recording a starting point, and the distance and compass bearing of a route around a property. Distances were typically measured in chains, and areas calculated in acres.

*Mete*: The act of metering, measuring, and assigning by measure (the units are usually feet or chains) a straight course.

*Bound*: The general property boundaries, such as those that follow a stream or road.

Boundaries of tracts are described verbally, beginning with some "point of beginning," and returning to this same point.

Parcels were usually shaped haphazardly, and described as such:

*Beginning at the mouth of a creek at an ash stump  
thence up the creek south 20 chains to a beach tree  
thence east 42 chains to a small walnut  
thence north 50 east 30 chains to a hickory  
thence north 38 chains to a creek  
thence west 20 chains with Johnson's line until it intersects  
with Mogg's line  
thence south 30 west 20 chains to a white oak ....*

## 2.2 ROYAL HEADRIGHT SYSTEM

During the colonization of Georgia, and until the American Revolution, land was distributed to settlers by grants and Headright warrants issued by trustees appointed by the British Crown. The Headright system is so named because it was a per-capita system of allocation of lands. Any qualified male citizen of Georgia could, on "land day," petition the royal governor for land. Each qualified individual was entitled to 100 acres of land, plus an extra 50 acres for each family member, indentured servant, or slave. The matter of issuing grants was a very serious affair, and the Surveyor General and his assistants were men of high standing and importance in colonial times, and certified and sworn to this duty.

In the beginning, surveyors attempted to keep tracts rectangular and preferably north-south in orientation. This, however, was difficult to achieve due to topography and politics. Most citizens wanted land in areas that were productive and easy to farm, such as the bottomland areas along major streams. At that point in time, water was the primary means of transportation. Since most people wanted water frontage, the length of tracts along streams was not supposed to exceed three times the tract width, with the short side along the river or stream. Royal land grants are recognized even today to convey ownership to the low water mark, while state grants only convey title to the high water mark.

In addition to the concerns of the early settlers, people wanted unclaimed land between them and their neighbors to remain unclaimed, so that they could graze their cattle on this "common" land. Individuals could physically identify the land that they wanted, but surveyors had to mark it out. As a result, surveyors were instructed to lay out tracts so that some good and some bad land was included in each tract, but political influence and fraud worked against these efforts.

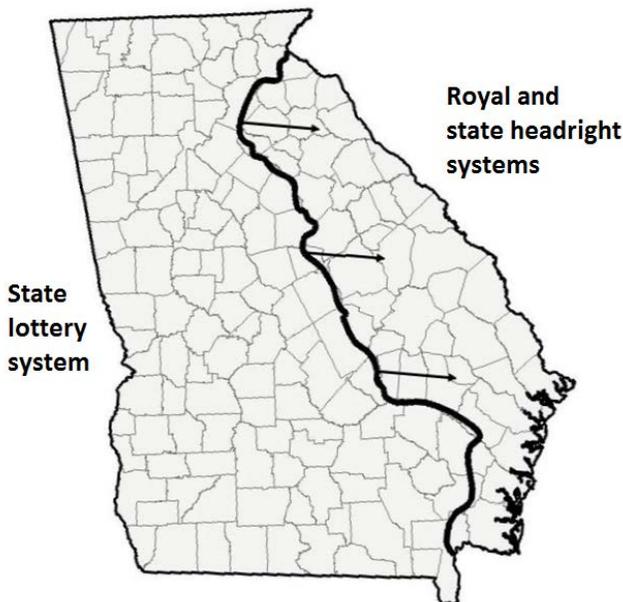
The last Royal Headright land grant on record was made in 1775.



## 2.3 STATE HEADRIGHT SYSTEM

In 1778, the newly constituted state government certified its first state surveyor. The basic headright philosophy was maintained, but now each head of household was allowed 200 acres, with an additional 50 acres for each family member or slave, up to a maximum of 10 people. No particular concern was made toward the shape of the tracts during this system of land division. For people considering the construction of a forge or a mill, 2,000 acres could be allowed.

This system became outdated with increases in population and the subsequent increase in demand for land. Disputes, claims, and lawsuits were commonplace, and still complicate some surveys today. Private settlements between parties were often not recorded in county courthouses or were lost through courthouse fires, and continue to confound surveyors today. The area covered by both headright systems is shown in the map below (from Cadle 1991).



## 2.4 STATE LOTTERY SYSTEM (1803 - 1831)

As time progressed in the settlement of Georgia, the demand for land increased. Under this system, as land was ceded to the U.S. from Indian tribes, the State partitioned the land into Georgia Militia Districts (GMD or District) which were originally formed in 1755 for local defense purposes. Each District had an elected Captain, and all able bodied males age 16 to 60 were organized into a military company. Initially, District lines were created, then lines were established at right angles to the District lines and Land Lots were delineated. Land Lots are square pieces of land, numbered consecutively.

When a land lottery occurred, people who qualified for land could enter, and lots were drawn by a state representative and awarded randomly. *Bounty grants* were special drawing rights in these lotteries granted to Revolutionary War veterans.

Land prices ranged from \$0.50 to \$9.00 per acre over the range of time that this system was in place. Married, qualified men and each widow were entitled to two draws from the lottery. Land Lot sizes decreased over time as population pressure increased, and as gold was discovered in north Georgia. In the beginning, Land Lots ranged in size from 200 to 490 acres.

Year	Size of land lots (acres)
1803	202.5 or 490.0
1806	202.5
1818	250.0 or 490.0
1819	250.0 or 490.0
1821	202.5
1825	202.5
1830	160.0
1831	40.0

At the end of the use of this system, land lots were 40 acres in size. However, some parts of southeast Georgia were surveyed last and land lot sizes there are 2,000 acres each. Most of the Okefenokee Swamp remains unsurveyed. There was a time when one could literally “get away with murder” because nobody knew where the County lines were and therefore no jurisdiction for trial could be established. Now of course, most of the Okefenokee is federally owned, so jurisdiction is firmly established.

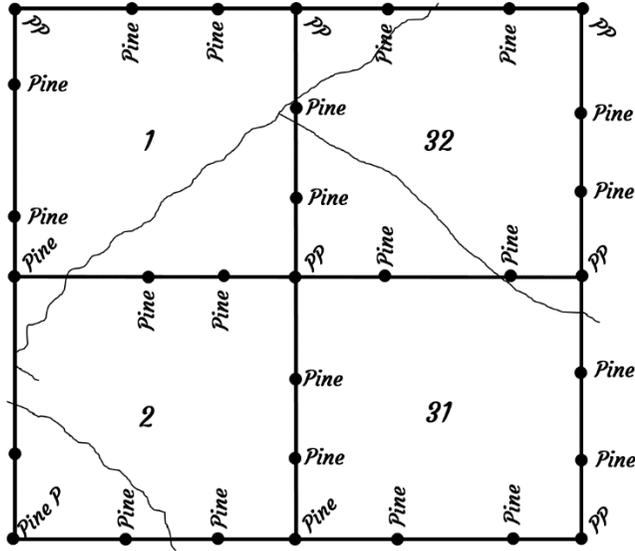
Although most of Georgia was surveyed under this system of land subdivision, a number of unforeseen problems were associated with it, including:

- Extensive fraud
- Surveying errors
- A lack of consistency in surveying processes (no reference origins, no baselines, no meridians)
- Hostility of the Indian tribes
- Remoteness of the terrain.

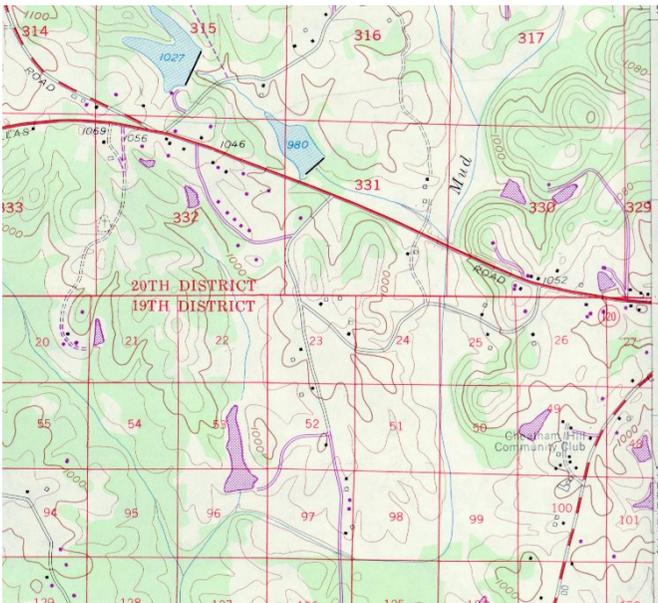
Shown on Page 12 is a rendering of a portion of District 16 of Dooly County, GA. Each lot is approximately 200 acres, and numbered consecutively north-south, and the lines are identified by tree species that are marked along the way.

Land Lots and their associated Districts can sometimes be found on USGS 7.5 minute quadrangle maps. Shown on Page 12 is a portion of two Districts where the sizes of the Land Lots are distinctly different.

### A rendering of the original survey of a portion of District 16, Dooly County, GA



### A USGS 7.5 minute quadrangle map containing land lots



## 2.5 PUBLIC LAND SURVEY SYSTEM

Established by the Land Ordinance of 1785 and the Northwest Ordinance of 1787, and managed by the U.S. General Land Office, the Public Land Survey System (PLSS) was designed as a way to survey and dispose of most the land in the U.S. beyond the original colonies. Both Florida and Alabama were surveyed using this system. Although the curvature of the earth prevents the delineation of a perfect square grid of land subdivision, the intent of the system was to develop Townships, Sections, and portions of Sections that were square or rectangular in nature. Most of western Canada was also surveyed under a similar system, the *Dominion Land Survey*.

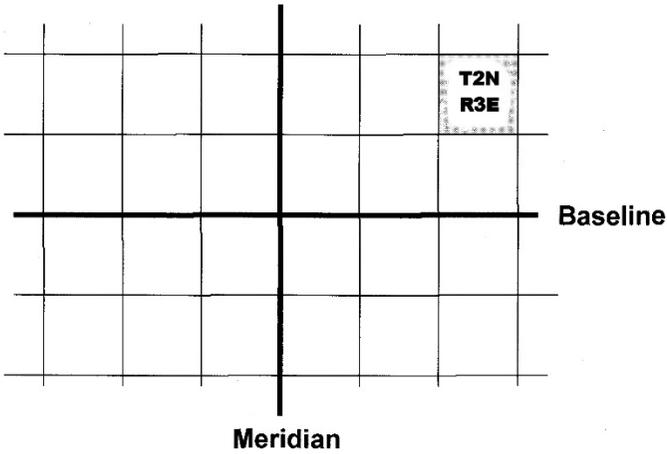
A baseline (east-west line) and a meridian (north-south line) for this system were established in almost every state. Some states, such as California, have more than one baseline and meridian pair. Some states (Oregon and Washington, for example) share a baseline and a meridian.

*Tracts* were initially surveyed beginning at the origin of each baseline and meridian. Tracts are 24 miles  $\times$  24 miles square. These were limited in size to account for changes in the landscape due to the curvature of the earth. Within each Tract, 16 *Townships* were delineated (4 Townships east-west, 4 Townships north-south).

A Township is thus 6  $\times$  6 miles square, and contains 36 square miles of land. Townships are identified by their relationship to the baseline and meridian. For example, T2N R3E indicates that a Township is 2 Townships north of a baseline, and 3 Townships east of a meridian.

Each Township, theoretically, was subdivided into 36 *Sections* of land that are 1 mile square. If a Section of land is 1 square mile, it is 80 chains wide  $\times$  80 chains tall, or 6,400 square chains. Thus each Section is 640 acres in size.

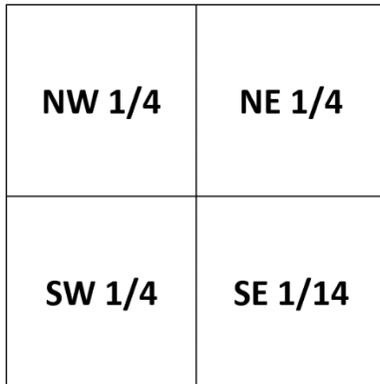
A Township, therefore, should theoretically contain 23,040 acres (36 Sections × 640 acres).



The 36 Sections within a Township are numbered in a serpentine manner. Section 1 is always in the upper-right corner, and Section 36 is always in the lower-right corner.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

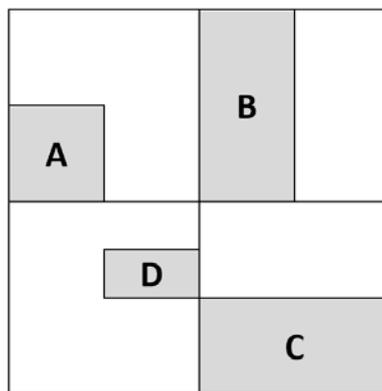
Sections are further subdivided into portions. The first subdivisions are the quarter-sections (NE, NW, SE, SW).



**Four quarters of  
Section 27  
(for example)**

Within each quarter-section, a Section can be further subdivided. In the figure below, Part A represents the SW 1/4 of the NW 1/4 of Section 27. If this were contained in T2N R3E, the full legal description of Part A would be:

SW 1/4 NW 1/4 Section 27 T2N R3E



**Section 27**

Part B represents the W 1/2 of the NE 1/4 of Section 27. The full legal description of this area of land is:

W 1/2 NE 1/4 Section 27 T2N R3E

**Example** What are the full legal descriptions for parts C and D in the example shown above?

Part C: S 1/2 SE 1/4 Section 27 T2N R3E

Part D: S 1/2 NE 1/4 SW 1/4 Section 27 T2N R3E

**Example** On the next page is an 1873 plat map of four Sections (19, 20, 29, and 30) of township T7N R19E near Pewaukee, Wisconsin. As you can see, each Section has been subdivided into smaller portions, many of them 40-acre pieces. How would you describe the 80-acre piece of land owned by E. Williams in Section 29?

W1/2 SW1/4 Section 29 T7N R19E

How would you describe the 40-acre piece of land owned by J. Benson in Section 20?

SE1/4 SE1/4 Section 20 T7N R19E

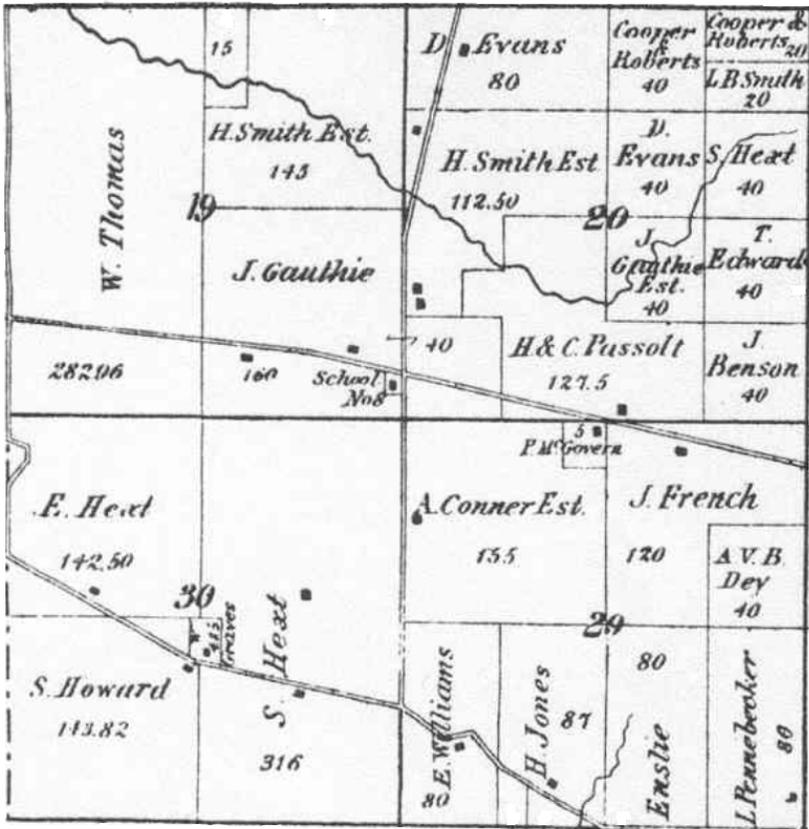
How would you describe the 160-acre piece of land owned by S. Howard in Section 30?

SW1/4 Section 30 T7N R19E

How would you describe the 20-acre piece of land owned by L.B. Smith in Section 20?

S1/2 NE1/4 NE1/4 Section 20 T7N R19E

**1873 plat map of Sections 19, 20, 29, and 30 of T7N R19E near Pewaukee, WI**



(Waukesha County Wisconsin Genealogy, 2011)

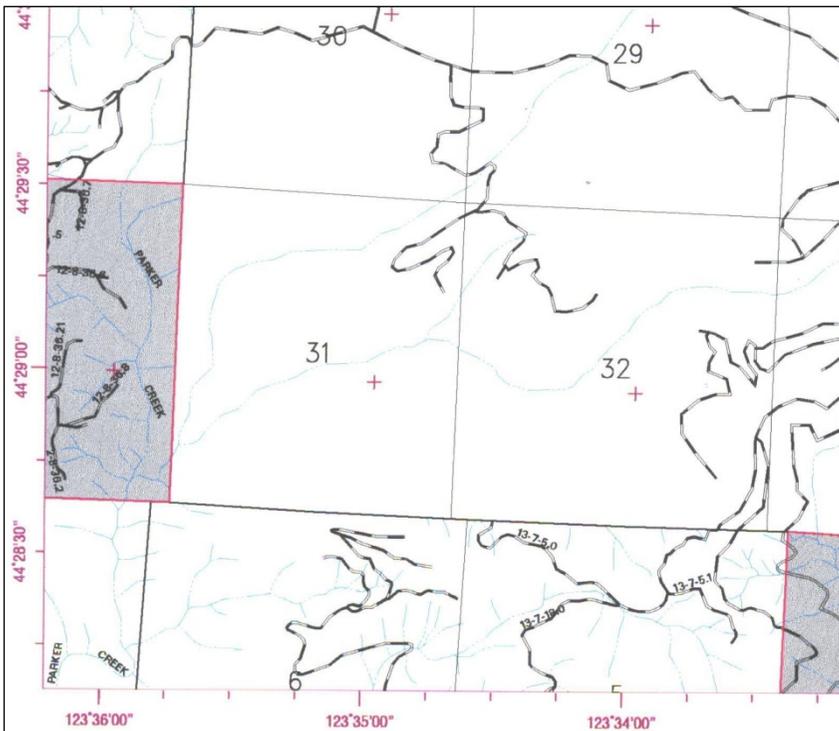
Important points

1. Townships were surveyed beginning with Section 36, and progressed northward to Section 1. After these six Sections were surveyed, the survey crew returned to Section 35, and surveyed the six Sections from there northward. This continued until all 36 Sections were completed.
2. As we noted earlier, due to the curvature of the earth, each

Township may not be exactly square (6 miles  $\times$  6 miles). Therefore, each Section within a Township may not be square.

3. Since Sections 1-6 were the last to be surveyed of each south-north line of Sections, and since Sections 6, 7, 18, 19, 30, and 31 were the last line of Sections surveyed, these 11 Sections may be irregular in shape (i.e., not square, and not 640 acres).

4. Crude instruments and inaccurate surveys led sometimes to the subdivision of land that was not quite as straight-forward as had been planned. See the map below. The Township which contains Sections 31 and 32 does not share edges or corners exactly as planned with other nearby Townships.



## **2.6 KNOWING WHERE YOU ARE - CORNERS AND BOUNDARIES**

To understand where you are on a tract, you could look for both tract corners and other evidence of property boundaries.

### Corners

Today, standard corner markings are iron pipes, perhaps 2.5" in diameter. In the PLSS system, some corners are represented by a brass cap that indicates the Section and Township.

In other surveying systems, corners are marked differently:

- Pits and mounds
- Blazed trees
- Stones with scribings on them
- Other improvisations like "lightard" knots.

### Evidence of boundaries

Boundaries of tracts are marked by blazes on trees, painted lines, signs, fences (or old fence posts and barbed wire), and changes in timber types.

When looking for evidence of tract boundaries, keep in mind that:

- They are not exact
- Most were located in the "vicinity" of the line
- Trees may have been blazed for other reasons
- Timber cutting may have not been up to property boundary.

Property boundaries are established by a professional surveyor, and the property lines are often mapped on a "plat."

## 2.7 COUNTY COURTHOUSE RECORDS

County courthouses have a Clerk's Office (or other similarly named office) where real property transactions are recorded as deeds, and are public records.

A *deed* is a legal instrument that is used to grant a right, such as the transfer of title to property from one person or entity to another. When considering real estate, a deed may convey the ownership of a property from one person (the *grantor*) to a new person (the *grantee*). A deed may convey the surface and subsurface (i.e., “mineral rights”) or it may convey just one of these rights.

A *warranty deed* conveys to the buyer title to a property. The seller guarantees (or warrants) that the title is good, and unencumbered, exactly as indicated in the deed. It often provides a promise that the product (in this case, land) is not defective, as should be expected by a reasonable buyer of the property. This is the most common type of deed for transferring real property, and requires a title search to be certain that the property is free and clear of any liens (i.e., whether the title is encumbered or “cloudy”). If the title is not clear, it must be “cured” before any prudent buyer will purchase it. Oftentimes, buyers are required by lenders to buy title insurance which will protect the lender if the title is later found to be faulty but not necessarily the buyer. Buyers or landowners may also buy owners’ title insurance which protects the owner if the title is faulty.

A *timber deed* conveys permanent ownership to standing timber on a property. Under English common law tradition adopted by the United States in 1791, standing timber is real estate. The timber deed expires when that timber is cut or dies.

A *tax deed* is the result of a tax sale. The buyer of the property must perform their own title search to determine whether the property is free and clear of all liens and encumbrances. A tax

deed is not a warranty deed.

A *quit claim deed* is a legal instrument whereby the seller of a property (or resource) quits any claim that they may have had to the property (or resource). This type of deed has the least assurance that the buyer has any right at all for the property being purchased. In fact, the seller makes no warranty that they actually own the property (or resource) in question.

Other kinds of deeds are *limited* or *special warranty* deeds, *voluntary* deeds, *security* deeds or *deeds to secure debt*, *executor's* deeds, *deeds of assent*, and *fiduciary* deeds.

Parts of a deed include:

- Grantor (seller)
- Grantee (buyer)
- Date of instrument
- Date of recording by the Clerk's Office
- A description of the property
- Reference to easements, rights of way, restrictions, etc.
- Reference to a plat or to a survey
- Testimonium, signature, and attestation

To locate a deed, you must refer to one of two indices.

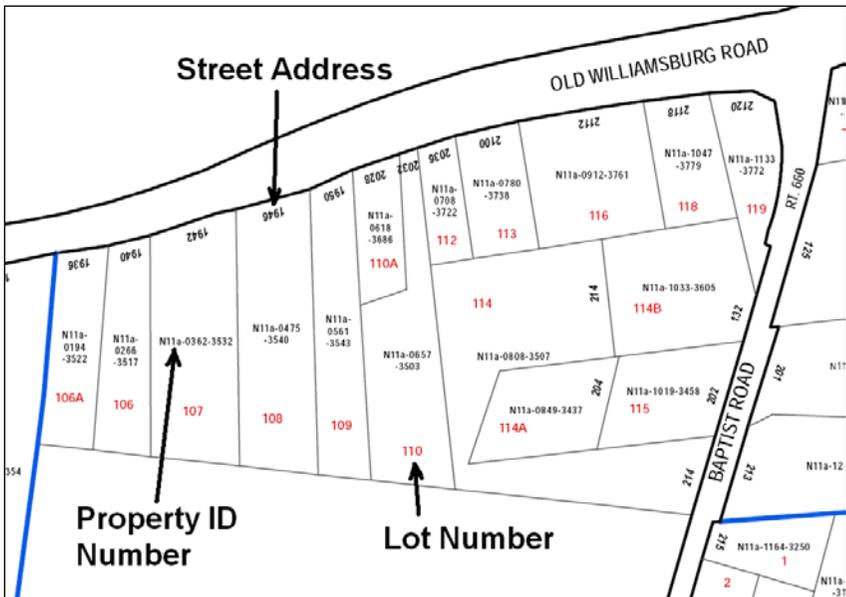
A *direct index* provides the location of a deed in the County Courthouse's set of deed books, where grantors are listed alphabetically using their last name (*Grantor Book*). A direct index is only useful for a specific period of time (such as a year) in which land transactions were recorded at the courthouse. Therefore, you may need to review numerous direct indices to determine when the grantor sold a particular piece of land.

A *reverse index* is similar to a direct index in that it provides the location of a deed in the County Courthouse's set of deed books (*Grantee Book*). The difference is that grantees (buyers) are listed alphabetically using their last name. As with a direct index, a



A *plat index* contains a list of the official surveys that were recorded at the County Courthouse. Some unrecorded plats and plats for minor subdivisions of land are not included in this index. A plat index contains the plat book and the page number for which one can locate a plat.

A *tax map* (see example below) can be obtained from most counties. Sometimes, ownership parcels are superimposed on aerial photographs to provide an enhanced view of property ownership with respect to the landscape. Properties are labeled by a property code number. Tax maps sometimes show acres and distances, and generally contain less survey-related data than plat maps. The tax map below was obtained from an Internet site in 2017 (York County, Virginia 2011), and provides the street address, lot number, and property identification number for each parcel.



(York County, Virginia 2011)

A *tax map index* provides a list of properties by their property code number and in some states by tax map and property parcel number. These may also be provided by street address of the property or by an alphabetical list of land owners. The index provides the owner, the owner's address, the property address, the size of the property, and the value of improvements.

Information provided on tax maps or by tax map indexes are not totally reliable. You need to refer to the current deed and official plat map to obtain the correct information. Many counties now provide tax maps through an Internet site. Check your county to determine what may be available in this manner.

### Important points

1. All records at County Clerk's Offices, except adoptions, are public information.
2. If you need assistance, ask the Clerk. While deeds and indices are similar in most states and counties, some variations on the system exist.
3. Copies of deeds, plats, or tax maps can be purchased from the Clerk's Office.
4. In Georgia, information on some deeds can be accessed through the Georgia Superior Court Clerk's Cooperative Authority ([www.gscca.org](http://www.gscca.org)), after registering with the system.
5. Some counties allow access to deeds on-line, provided you know the book and page number in which the deed was recorded. An example of a deed from Bibb County is provided on the next two pages.

Return Recorded Document to:  
Theodore T. Carellas, P.C.  
440 Silverwood Centre Drive  
Post Office Box 2599  
Rincon, GA 31326



Doc ID: 00987300002 Type: GLR  
 Filed: 10/19/2004 at 08:30:39 AM  
 Fee Amt: \$404.50 Page 1 of 2  
 Transfer Tax: \$484.50  
 Bibb County Superior Court  
 Dianne Brannen Clerk  
**BK 6375 PG 376-377**

**LIMITED  
WARRANTY DEED**

STATE OF GEORGIA  
COUNTY OF

FILE #: 0963-04

**THIS INDENTURE** made this 8th day of October, 2004, between Georgia Land & Timber, Inc. of the County of and State of Georgia, as party or parties of the first part, hereinafter called Grantor, and Bibb Classic Properties LLC as party or parties of the second part, hereinafter called Grantee (the words "Grantor" and "Grantee" to include their respective heirs, successors and assigns where the context requires or permits).

**WITNESSETH** that: Grantor, for and in consideration of the sum of TEN AND NO/100 DOLLARS (\$10.00) and other good and valuable considerations in hand paid, at and before the sealing and delivery of these presents, the receipt whereof is hereby acknowledged, has granted, bargained, sold, aliened, conveyed and confirmed, and by these presents does grant, bargain, sell, alien, convey and confirm unto the said Grantee.

All that tract or parcel of land situate, lying and being in Land Lot 64, of the 7<sup>th</sup> Land District of Bibb County, Georgia, and being more particularly described as a tract of land containing 167 acres, more or less, and being all of that portion of said Land Lot 64 which lies west of Stone Creek and being the same property conveyed W. A. Flagg by deeds recorded in Deed Book 455, page 99, Deed Book 113, page 772, Deed Book 138, page 679, Deed Book 138, page 678, Deed Book 126, page 799, and Deed Book 119, page 288, in the office of the Clerk of Superior Court of Bibb County, Georgia.

This being the same property conveyed by Receiver's Deed to Georgia Land & Timber, Inc., dated January 23, 2004, recorded in Deed Book 6079, page 274, aforesaid records.

This Deed is given subject to all easements and restrictions of record.

**TO HAVE AND TO HOLD** the said tract or parcel of land, together with all and singular the rights, members and appurtenances thereof, to the same being, belonging or in anywise appertaining, to the only proper use, benefit and behoof of the said Grantee, forever in **FEE SIMPLE**.

**AND THE SAID** Grantor will warrant and forever defend the right and title to the above described property unto the said Grantee against the claims of all persons by, through and under the above named grantor.

**IN WITNESS WHEREOF**, Grantor has hereunto set grantor's hand and seal this day and year first above written.

Signed, sealed and delivered in presence of:

Witness 

Notary Public 



Georgia Land & Timber, Inc. 

Larry Brian Ballard 

PT-61 011-2004-000205

SECTION A - SELLER'S INFORMATION (Do not use agent's information)		SECTION C - TAX COMPUTATION	
SELLER'S BUSINESS NAME Georgia Land & Timber, Inc.		Exempt Code If no exempt code enter NONE	NONE
MAILING ADDRESS (STREET & NUMBER) P. O. Box 7151		1. Actual Value of consideration received by seller Complete Line 1A if actual value unknown	\$484,418.00
CITY, STATE, ZIP CODE Macon, GA 31201	DATE OF SALE 10/8/2004	1A. Estimated fair market value of Real and Personal property	\$0.00
SECTION B - BUYER'S INFORMATION (Do not use agent's information)		2. Fair market value of Personal Property only	\$0.00
BUYER'S BUSINESS NAME Bibb Classic Properties LLC		3. Amount of liens and encumbrances not removed by transfer	\$0.00
MAILING ADDRESS (Must use buyer's address for tax billing & notice purposes) 440 Silverwood Centre Drive		4. Net Taxable Value (Line 1 or 1A less Lines 2 and 3)	\$484,418.00
CITY, STATE, ZIP CODE Rincon, GA 31326	Check Buyers Intended Use ( ) Residential ( ) Commercial ( ) Agricultural ( ) Industrial	5. TAX DUE at 10 per \$100 or fraction thereof (Minimum \$1.00)	\$484.50
SECTION D - PROPERTY INFORMATION (Location of Property (Street, Route, Hwy, etc))			
HOUSE NUMBER & EXTENSION (ex 285A)	PRE-DIRECTION, STREET NAME AND TYPE, POST DIRECTION		SUITE NUMBER
COUNTY BIBB	CITY (IF APPLICABLE)	MAP & PARCEL NUMBER 28-100	ACCOUNT NUMBER
TAX DISTRICT	GMD	LAND DISTRICT 7th	ACRES 167
		LAND LOT Lot 64	SUB LOT & BLOCK
SECTION E - RECORDING INFORMATION (Official Use Only)			
DATE 10/19/04	DEED BOOK 6375	DEED PAGE 376	PLAT BOOK
			PLAT PAGE

ADDITIONAL BUYERS  
None

## 2.8 REFERENCES

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Waukesha County Wisconsin Genealogy. 2011. Plat maps for Waukesha County townships. Waukesha County Wisconsin Genealogy, Waukesha, WI.  
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York County, Virginia. 2011. Tax maps. Office of Geographic Information, York County, VA.  
<http://www.yorkcounty.gov/CountyGovernment/InformationTechnology/GISMapping/TaxMaps.aspx> (accessed 11/21/19).

### Other sources of information include

DeVorse, L. Jr. (ed.). 1971. DeBrahm's report of the general survey in the southern district of North America. Tricentennial Edition, Number 3, University of South Carolina Press, Columbia, SC. 325 p.

*Georgia v. Ashmore* 236 Ga. 401 (1976).

Hemperly, M.R., and E. Jackson. 1993. Georgia's boundaries: The shaping of a state. Carl Vinson Institute of Government, University of Georgia, Athens, GA. 116 p.

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- Linklater, A. 2003. *Measuring America: How the United States was shaped by the greatest land sale in history*. Plume, New York, NY. 311 p.
- Mays, L.B. 1975. *Settlers of the Okefenokee: Seven biographical sketches*. Okefenokee Press, Folkston, GA. 186 p.
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### 3.0 LAND MEASUREMENTS

Measuring land and navigating across the land (through the woods, across fields, etc.) is a skill that all natural resource managers need to obtain.

While natural resource managers may not be responsible for surveying property boundaries, they often need to locate the boundaries, navigate across the landscape along sample lines, and map various landscape features (streams, timber stands, wildlife habitat). To be able to perform these tasks, you need to become familiar with the common approaches for measuring land areas in the field with measuring tapes and by pacing, and to measure directions using a compass. Mapping land areas by hand requires a sense of scale and attention to detail when drawing distances and directions. Measuring the size of land areas requires knowledge of one or more land area computation methods.

After completing this Chapter, you will be able to:

1. Understand scale, and how ground distances relate to mapped distances.
2. Understand how to effectively use an Engineer's scale.
3. Understand how to measure horizontal distances on the ground using tapes or your pacing skills.
4. Understand how to use a compass, how declination is set, and how to convert between azimuths and bearings.
5. Understand how to assess the quality of traverse measurement field work by computing interior and deflection angles of a traverse.
6. Understand the issue of error of closure of a traverse, and how it can be corrected.
7. Understand how to make a map from field measurements.
8. Understand how to calculate the area of a closed traverse.

### 3.1 MEASURING DISTANCES ON MAPS

Maps are smaller representations of a larger landscape. Therefore each map should have associated with it a scale. The scale of a map helps you estimate actual ground distance between two or more points. Scales can be presented as graphical representations, as a representative fraction, or as an equivalence.

Graphical scale:



Representative fraction scale: 1:24,000

Equivalence scale: 1" = 10 chains

Notice with the graphical scale that one would need to physically compare a distance on a map to the image of the scale itself to determine distance. A scale bar could be used to develop a rough relationship between the graphical scale and distances on a map.

The representative fraction scale is unitless. When using this scale, 1 unit of *something* measured on a map (an inch, a centimeter, etc.) equals 24,000 (in this case) of those *same units* on the ground. Therefore, if you measure 1 inch on a 1:24,000 scale map, that one inch represents 24,000 inches on the ground.

The equivalence scale has different units of measurement on either side of the scale. To interpret the scale above, you would assume that 1 inch measured on a map represents 10 chains on the ground.

#### Ground distance and map distance

To develop a scale, you need two measurements: ground distance and map distance. Ground distance is the actual distance measured on the ground. Map distance is the distance on a map

covering the same area. A scale is simply the relationship between the two measurements.

**Example** You measure on a map a straight line between two road intersections and find it to be 4.3 inches. In the field, you measure the same straight-line distance between the two road intersections, and find it to be 86 chains. What are the equivalence and representative fraction scales?

The equivalence scale:

4.3 inches = 86 chains            (we want "1 inch" on the left-hand side, so divide each side by 4.3)

1 inch = 20 chains

The representative fraction scale:

Since this scale needs to be unitless, we need to convert one of the two sides of the equivalence scale to the other side's units:

$$1 \text{ inch} = 20 \text{ chains} \left( \frac{66 \text{ feet}}{\text{chain}} \right) \left( \frac{12 \text{ inches}}{\text{foot}} \right)$$

1 inch = 15,840 inches, or 1:15,840

**Example** The scale of a map is 1:24,000. You measure a distance on the map (3.21 inches). How far, in feet, is this on the ground?

When using a representative fraction scale such as 1:24,000, you should say to yourself "1 unit on the map equals 24,000 of the same units on the ground." If the units are inches, then each map inch represents 24,000 inches on the ground. Since you measured 3.21 map inches, the ground distance is  $3.21 \times 24,000$ , or 77,040 ground inches, or 6,420 feet.

### 3.2 USING AN ENGINEER'S SCALE

An Engineer's scale is a triangular ruler that includes 6 different scaling systems.

When you hold the scale in your hand, you will see on the left-hand side a number (10, 20, 30, 40, 50, or 60). You can see three of these by rotating the scale. To see the other three, flip the scale left-to-right, then rotate it.

The scale numbers (10, 20, 30, 40, 50, or 60) represent *parts per inch*.

Therefore, when using the 60 scale, each hash mark represents 1/60 of an inch. This is the finest scale available on the Engineer's scale, and allows you to develop some very precise measurements.

In contrast, when using the 10 scale, each hash mark represents 1/10 of an inch. The advantage of using the 10 scale is that the gradations 1 through 12 represent actual inches.

For scales 20, 30, 40, 50, and 60, you must add a "0" to each gradation to understand how many "parts" there are. For example, the "24" on the 60 scale actually represents 240 parts. If each part is 1/60 of an inch, the distance from 0 to 240 parts is:

$$\text{inches} = \left( \frac{\text{parts measured}}{\text{scale}} \right) \text{ or, } \left( \frac{240}{60} \right) = 4 \text{ inches}$$

**Example** You measure a distance on a map using the 50 scale. It is 225 parts. What is the map distance?

$$\text{inches} = \left( \frac{\text{parts measured}}{\text{scale}} \right) = \left( \frac{225}{50} \right) = 4.5 \text{ inches}$$

### 3.3 MEASURING DISTANCES ON THE GROUND

There are several ways to measure distances on the ground:

- Measure the distance with a measuring tape
- Record the beginning and ending coordinates provided by a GPS receiver and use the Pythagorean theorem to determine distance
- Pace the distance

#### What are you measuring?

You need to be clear about one thing - maps provide the horizontal distance between two points, as if you were looking down on the landscape from an airplane. And, when drawing a map, you use horizontal distances to do so.

When you physically measure distances on the ground, and you relate these to a map, you need to arrive at a horizontal distance. This is difficult, because some land can slope significantly uphill or downhill (depending on your travel route).

So, you can either

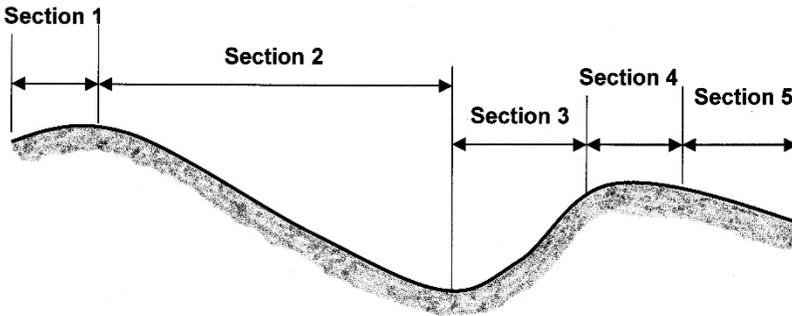
1. Measure the horizontal distance on the ground, or
2. Measure the slope distance and convert it to the horizontal distance.

### 3.4 MEASURING HORIZONTAL DISTANCES WITH A MEASURING TAPE

To measure the horizontal distance between two points using a measuring tape, the tape must remain perfectly level.

If you are working in areas with significant slopes, a transect will need to be broken down into sections, and each section will need to be measured separately.

The distance between sections will depend on the slope of the ground. Obviously, this is a two-person operation, and one person will be holding one end of the tape near the ground surface while the other person holds the tape in the air - so that the tape remains perfectly horizontal. The amount of distance the team can cover on each section may vary.



Once completed, the transect will be comprised of measurements that represent horizontal distances, thus they do not need to be converted from slope distances to horizontal distances prior to drawing a map.

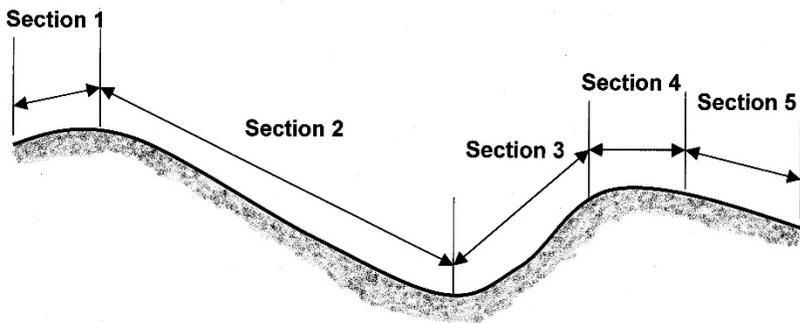
#### Important point

1. A plumb-bob may be necessary to locate one end of a section (the end where the tape is held in the air).

### 3.5 MEASURING SLOPE DISTANCES WITH A MEASURING TAPE

To measure the slope distance between two points using a measuring tape, the tape need not remain perfectly horizontal. In fact, the tape should be held at the same height above ground by both people involved in collecting the measurements.

If you are working in areas with significant slope or broken terrain, a transect will need to be broken down into sections, and each section will need to be measured separately.



Once completed, the transect will be comprised of measurements that represent slope distances, thus they need to be converted from slope distances to horizontal distances prior to drawing a map. The conversion from slope distance to horizontal distance uses trigonometry.

Horizontal distance = (slope distance) (cosine of slope ( $^{\circ}$ ))

**Example** You measure a slope distance along a portion of transect. It is 75 feet. Slope is calculated to be  $17^{\circ}$ . What is the horizontal distance?

Horizontal distance = (75 feet) (cos ( $17^{\circ}$ )) = 71.7 feet

If you collect the slope as a percent measurement (e.g., 30

percent), you need to convert it to "decimal percent" (e.g., 0.30) and then convert this value to degrees using the following formula:

$$\text{Slope in degrees} = \tan^{-1}(\text{slope in decimal percent})$$

**Example** You measured a portion of a transect. The length is 125 feet, the slope is 32%. What is the horizontal distance of this side of the transect?

$$\text{Slope in degrees} = \tan^{-1}(0.32) = 17.74^\circ$$

$$\text{Horizontal distance} = (125 \text{ feet}) (\cos (17.74^\circ)) = 119.1 \text{ feet}$$

### Important points

1. Your calculator may be in a mode where the cosine function expects values in radians. You may need to change the mode of your calculator to accept degrees in the cosine function.
2. "Slope" can be viewed as an angle (e.g.,  $16.7^\circ$ ) or as a percent (e.g., 30%). When slope is described from the *percent* perspective, it can also be viewed as the "rise over the run." The "rise" is the change in elevation from one end of a line to the other. The "run" is the length of the line. Therefore,

$$\text{Slope in percent} = \left( \frac{\text{rise}}{\text{run}} \right) \text{ or } \left( \frac{\text{change in elevation}}{\text{length of line}} \right)$$

Of course, for this equation to work, both measurements (rise and run) need to be in the same units.

3. It may be more reasonable to measure horizontal distances rather than slope distances due to potential errors that may propagate in the measurements and subsequent calculations, and their potential impacts on the map that is being produced.

### 3.6 PACING DISTANCES

One of the most common ways for natural resource managers to estimate distances is to pace them.

A *pace* can either be every step, or every other step. You need to decide this and stick to a system.

A *pace count* is the number of paces per chain.

Generally, when we pace, we attempt to walk the horizontal distance (even though the transect may be on a slope).

Since walking in the woods (over down logs, through briars, etc.) is different than walking in a grassy field, and since walking up or down a hill is different than walking on level ground, you need to understand the pace that is needed for each situation.

My pace count for level, open ground is \_\_\_\_\_ per chain.

My pace count for level, forested areas is \_\_\_\_\_ per chain.

My pace count for downhill, forested areas is \_\_\_\_\_ per chain.

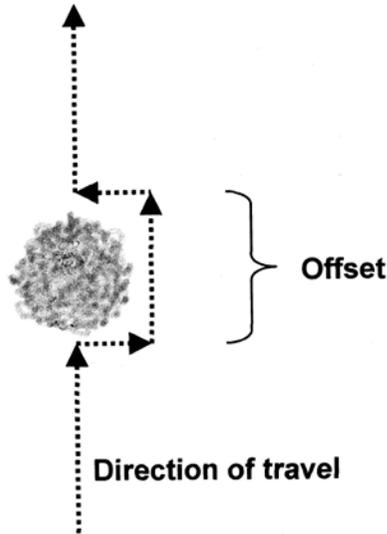
My pace count for uphill, forested areas is \_\_\_\_\_ per chain.

#### Important points

1. When pacing, walk normally. Don't be deliberate in stepping from one foot to the other while pacing.
2. Practice often on a 1-chain test course that you can set up yourself using a measuring tape.
3. Trust your work. Don't let the pacing effort of others affect your distance measurements - they may be wrong.
4. If you feel a need to avoid an obstacle, use an off-set. An off-set is a right-angle diversion. For example, you may turn 90° to

the right, go 1/2 chain, then turn forward and pace a chain, then turn 90° to the left and go 1/2 chain.

When using an off-set, you can avoid an obstacle (such as the briar thicket shown below), but more importantly you can end up about where you should have been had you simply paced forward through the obstacle.



### 3.7 USING A COMPASS

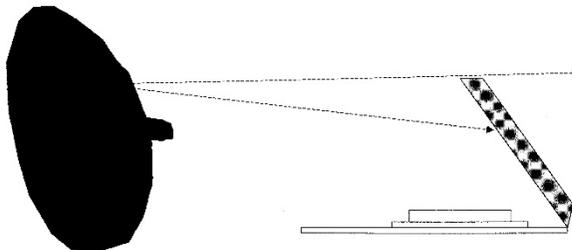
The mirror compass, such as the Silva Ranger, is the type that we recommend for navigation in the field and for rough mapping of transects and boundaries of stands.



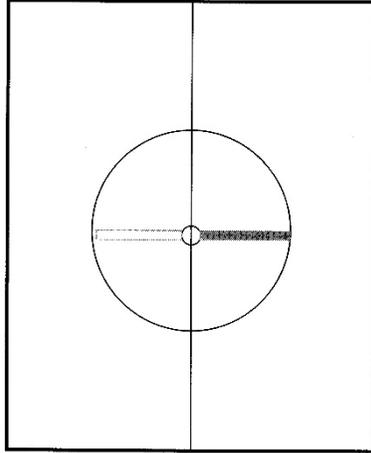
Compasses provide either with azimuth readings (0-360°) or bearing readings (0-90° in four quadrants), or both. We concentrate on the azimuth readings here.

To use the compass correctly:

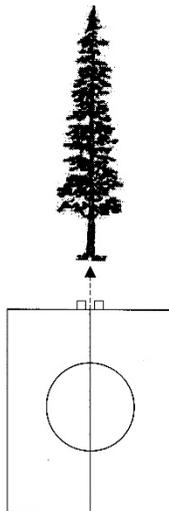
1. Hold the compass just below eye level, with the black cover (lid) that contains the mirror open. You need to be able to both look over the compass lid and look into the compass mirror.



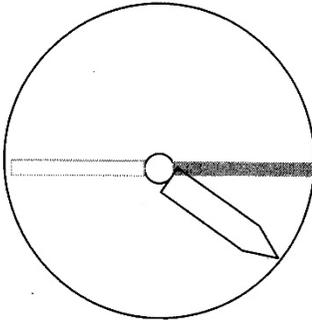
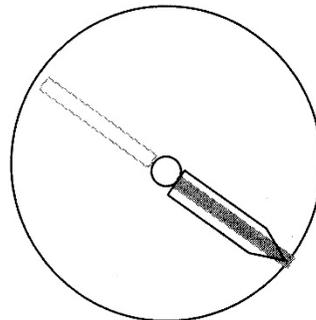
2. While looking into the mirror, align the vertical line (found in the center of the mirror) with the center of the compass needle. What you should see in the mirror is something like this:



3. Using the sight at the top of the lid of the compass, adjust yourself so that the target or direction you are heading can be seen directly through the sight.

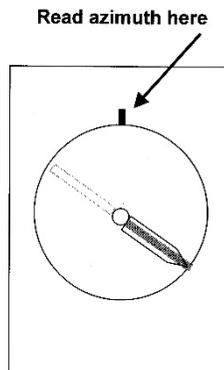


4. With your free hand, carefully turn the dial that contains the azimuth readings until the red side of the floating needle is inside the red outlined arrow (as some people say, "put red in the shed"). All of this is done while looking at the dial through the mirror.

**Before****After**

5. Make sure that the base of the compass is level (not tipped forward, backward, or to the a side), and adjust it as necessary.

6. When done, open the compass and read the azimuth at the hash mark located at the top of the compass dial.



### Important points

1. If you are using the compass to measure the direction

(azimuth) of a side of a traverse, sight from one corner to the next to obtain the azimuth associated with the side of the traverse.

The *foresight* is the azimuth in the direction that the survey is going, from one starting corner to an ending corner.

The *backsight* is the azimuth associated with the opposite direction, from the ending corner back to the starting corner. The backsight should be, ideally,  $180^\circ$  opposite of the foresight. The backsight should be taken to check the foresight measurement. If they are more than a few degrees (of  $180^\circ$ ) off, they need to be remeasured.

**Example** A foresight of  $37^\circ$  should have a backsight of what?  $217^\circ$  ( $37^\circ + 180^\circ$ ).

**Example** A foresight of  $190^\circ$  should have a backsight of what?  $10^\circ$  ( $180^\circ - 90^\circ$ ).

- If the foresight is  $179^\circ$  or less, add  $180^\circ$  to it to arrive at the backsight.
- If the foresight is  $180^\circ$  or more, subtract  $180^\circ$  from it to arrive at the backsight.

2. If you are using a compass to navigate through the woods, turn the dial so that the direction you need to head is aligned with hash mark located at the top of the compass dial (Step 6). Then, orient your body so that the red side of the floating needle is inside the red outlined arrow (Step 4).

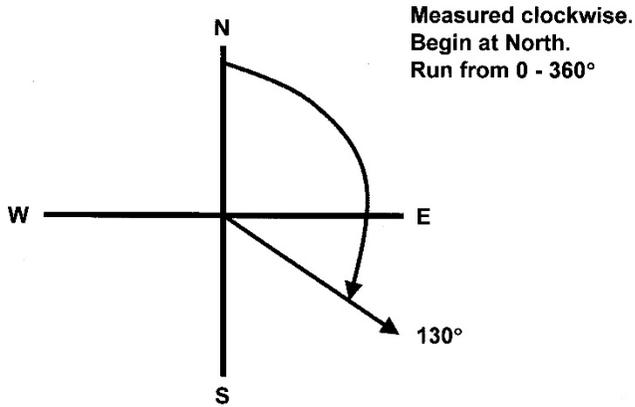
3. Most errors in compass measurement arise from not holding the compass level.

4. Small bubbles inside the compass capsule are not a problem. If a compass contains larger bubbles, it should not be used.

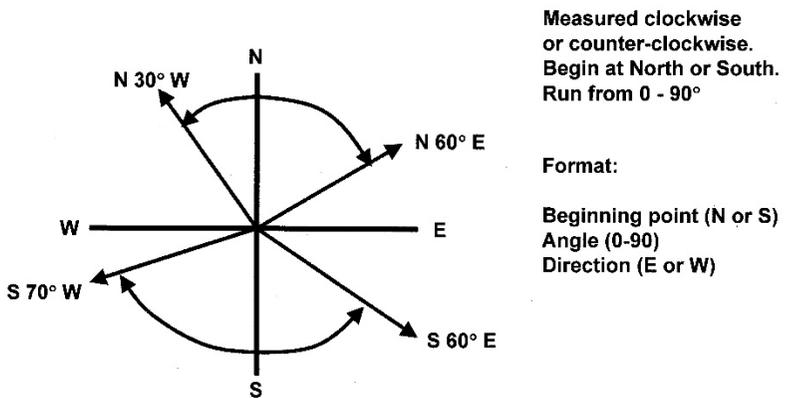
### 3.8 AZIMUTH AND BEARING READINGS

The two general types of directional measurements we make in natural resource management are azimuths or bearings.

Azimuths are measured clockwise from North, which is 0° (or 360°). There are 360° associated with this system.



Bearings are measured either from the North or from the South, and go to a maximum of 90° to the east or to the west. No angle greater than 90° should be associated with a bearing.



Bearings are presented as a value with three pieces of information:

The beginning point (North or South)

The angle ( $0-90^\circ$ )

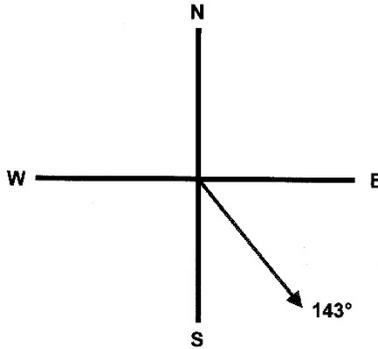
The direction headed from the beginning point (West or East)

**Example**  $N\ 32^\circ\ E$  represents a bearing that begins at North, and is directed  $32^\circ$  towards the East.

**Example**  $S\ 54^\circ\ W$  represents a bearing that begins at South, and is directed  $54^\circ$  towards the West.

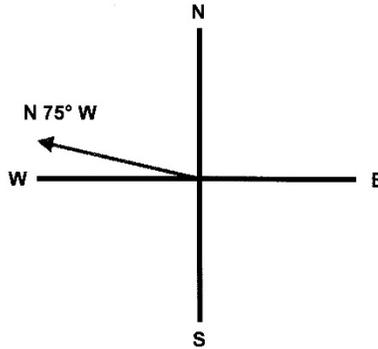
You can convert azimuths to bearings (or vice versa) very easily if you can visualize a compass in your mind.

**Example** What is the bearing associated with a  $143^\circ$  azimuth?



As you can see, if this were a bearing, it would have started from the South, and is directed towards the East. Since South has a  $180^\circ$  azimuth value, the angle is  $37^\circ$  from South ( $180^\circ - 143^\circ$ ). Thus the bearing associated with a  $143^\circ$  azimuth is  $S\ 37^\circ\ E$ .

**Example** What is the azimuth associated with a  $N\ 75^\circ\ W$  bearing?



If this were an azimuth, its value would be greater than  $270^\circ$ , which represents West. So, it is somewhere between  $270^\circ$  and  $360^\circ$ . In fact, it is  $75^\circ$  from  $360^\circ$ , since the bearing starts from North, and is directed  $75^\circ$  towards the West. Alternatively, it is  $15^\circ$  from West, since there are only  $90^\circ$  in each quadrant. Therefore, the azimuth is  $285^\circ$ , which could be found with one of two calculations:

$$\text{azimuth} = 270^\circ + 15^\circ$$

$$\text{azimuth} = 360^\circ - 75^\circ$$

**Example** What is the azimuth associated with a  $S 25^\circ W$  bearing?  
 $205^\circ$

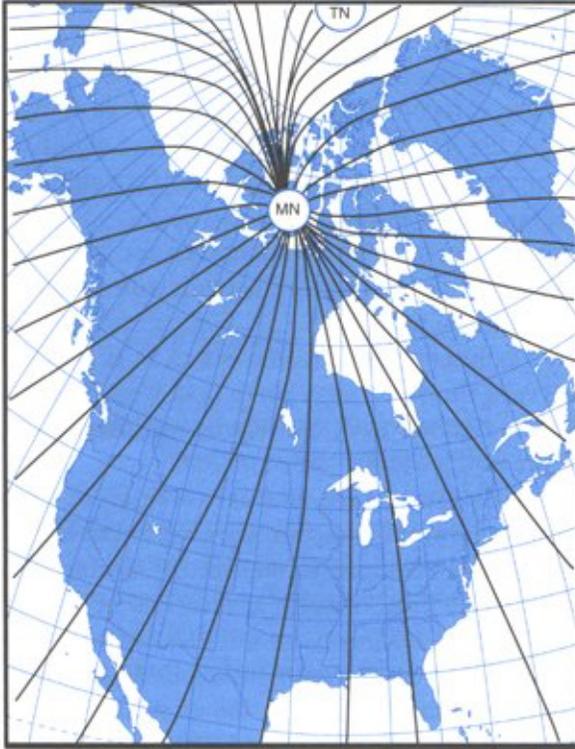
**Example** What is the azimuth associated with a  $N 75^\circ E$  bearing?  
 $75^\circ$

**Example** What is the bearing associated with a  $249^\circ$  azimuth?  
 $S 69^\circ W$ .

**Example** What is the bearing associated with a  $3^\circ$  azimuth?  
 $N 3^\circ E$

### 3.9 COMPASS DECLINATION

Declination is the angular difference between the magnetic meridian and the true meridian, or the difference between the magnetic North and the true North. Your compass points toward the magnetic North, not the North Pole (true North).



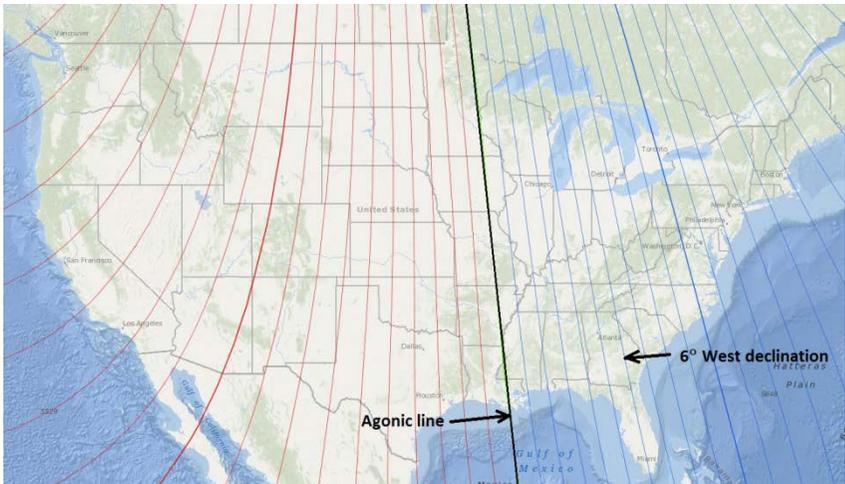
U.S. Centennial of Flight Commission (2000)

If your compass, when pointing to the magnetic North (MN) happens to also be pointing to the east of the true North (TN), the declination is said to be EAST.

If your compass, when pointing to the magnetic North happens to also be pointing to the west of the true North, the declination is said to be WEST.

The agonic line is the line from North to South where declination is  $0^\circ$ . Currently, the agonic line runs through an area west of Baton Rouge, MS.

For places West of the Agonic Line, declination is EAST.  
For places East of the Agonic Line, declination is WEST.



(National Geophysical Data Center 2015)

If you are using a compass that does not allow you to set the declination, or if you have the declination set at  $0^\circ$ ,

- a) East declinations are added to each magnetic measurement (field measurement that is oriented towards the magnetic north) to arrive at the true azimuth.
- b) West declinations are subtracted from each magnetic measurement (field measurement that is oriented towards the magnetic north) to arrive at the true azimuth.

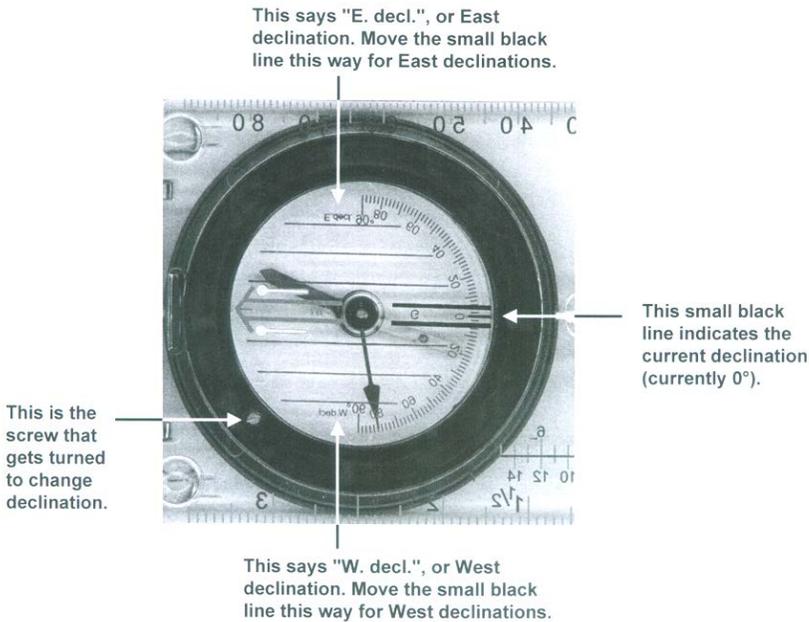
**Example** Assume you are working around Athens, GA, and you recorded these magnetic measurements with declination set to 0° on your compass:

45°, 120°, and 330°

What are the true azimuths if declination is assumed 6° West?

39°, 114°, and 324°

The current declination for Athens, GA is 5°55' W, or about 6° W. To change the declination on a Silva Ranger compass, look for the screw on the back side of the base of the compass, and using the small screwdriver that is attached to the tether (lanyard), turn the screw until the declination setting is 6° W.



### Important points

1. Declination is not constant, it changes over time.
2. A compass may also be affected by "local attraction." Local attraction includes areas with iron ore deposits, objects of iron and steel (such as railroad tracks, vehicles, watches, pens), and electric distribution lines. Take both forward and back readings of angles to ensure that local attraction is not significant.
3. To estimate the current declination for any location in the United States, the following National Geophysical Data Center (2011) website can be used:

<https://www.ngdc.noaa.gov/geomag-web/#declination>

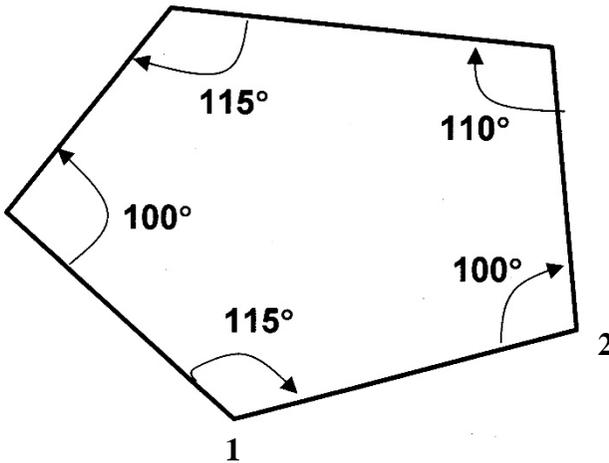
### 3.10 INTERIOR AND DEFLECTION ANGLES

When assessing the quality of field work conducted to map a closed area, one way to check the quality of your compass work is to compute the sum of the interior angles of the closed area. The sum of the interior angles inside a closed area is:

$$\text{Sum of interior angles (degrees)} = (\text{number of sides} - 2)(180)$$

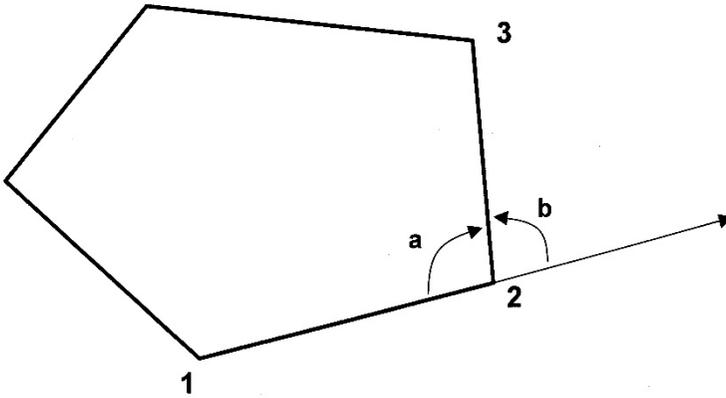
**Example** In the figure below, the interior angles were determined for a 5-sided area. The sum of the interior angles should be

$$\text{Sum of interior angles (degrees)} = (5 - 2)(180) = 540$$



How were the interior angles determined?

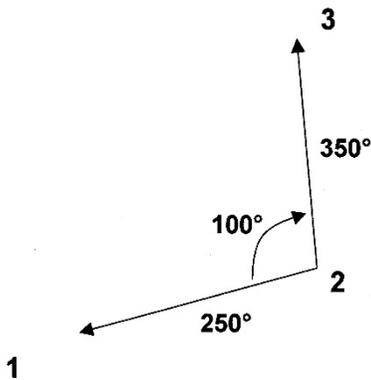
Assume that a survey began at point 1, and was conducted in the direction of point 2. If a line was extended straight beyond point 2, it forms a  $180^\circ$  angle. The angle inside the tract (a) is the interior angle. The angle outside the tract (b) is the deflection angle.



To visualize how to determine the interior angle (a), you need:

1. The backsight of the azimuth from the direction you came (from point 2 to point 1).
2. The foresight of the azimuth to the direction in which you will be going (from point 2 to point 3).

Assume that the direction from point 1 to point 2 had an foresight azimuth of  $70^\circ$ , and the foresight azimuth from point 2 to point 3 was  $350^\circ$ . The backsight azimuth from point 2 to point 1 should be  $250^\circ$ .



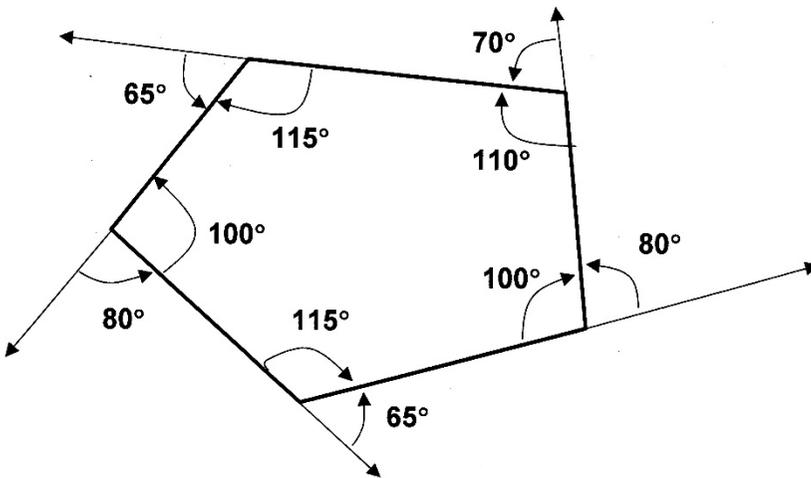
You can now calculate the interior angle by taking the difference between the two angles that project away from point 2.

$$\text{Interior angle} = (350^\circ - 250^\circ) = 100^\circ$$

A deflection angle is the angle outside the tract that is deflected from the straight line prolonged through the occupied station (or point). In the case of point 2, since the interior angle was  $100^\circ$ , the deflection angle must be  $80^\circ$ , because together they must add up to  $180^\circ$ .

Deflection angles are designated "R" or "L" depending on whether the survey turns to the right or the left of the prolonged line that is extended through the occupied point. In the case above, since you are travelling from point 1 to 2 to 3, and thus turning left at point 2, the deflection angle at point 2 is  $80^\circ$  L.

The other interior and deflection angles can be determined in a similar manner.

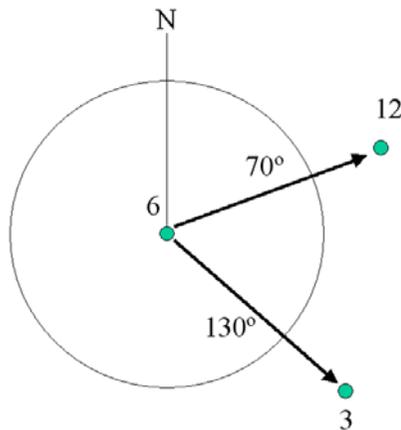


Contrary to the sum of the interior angles, deflection angles always should sum to  $360^\circ$ .

When determining interior angles, there are at least 3 "cases" that should be kept in mind.

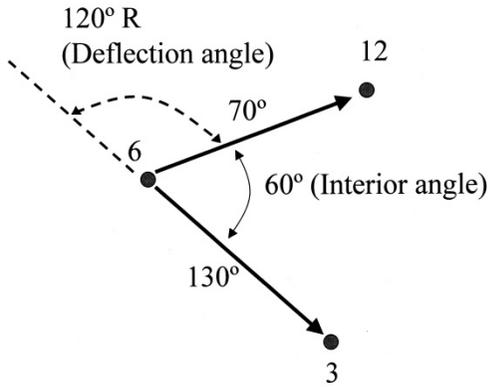
CASE 1 - North is not "involved" and the interior angle is less than  $180^\circ$ .

If the direction of travel in the example below was from point 3 to point 6, and the foresight azimuth from point 3 to point 6 was  $310^\circ$ , the backsight azimuth from point 6 to point 3 should be  $130^\circ$ . Here, "north" is not inside the interior angle.



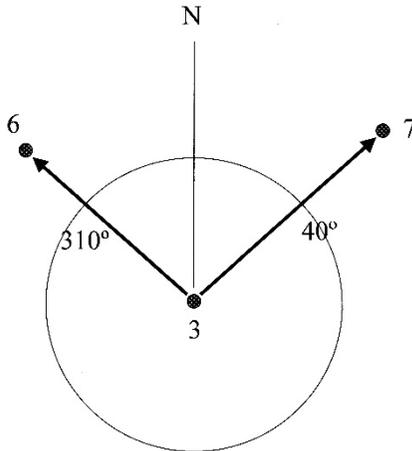
The interior angle about point 6 is then the larger of the two azimuths projected from it minus the smaller of the two azimuths, or  $60^\circ$ . The deflection angle is  $120^\circ$ , since the total of the interior and deflection angles about a station (point) should sum to  $180^\circ$ .

$$\text{Interior angle} + \text{deflection angle} = 180^\circ$$



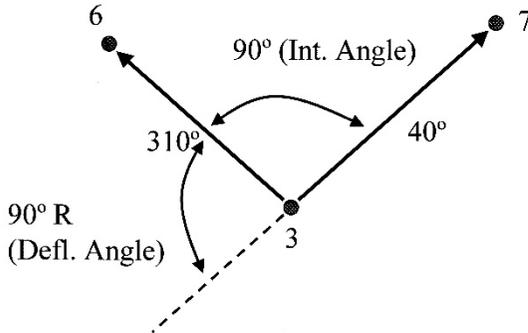
CASE 2 - North is "involved" and the interior angle is less than 180°.

If the direction of travel in the example below was from point 7 to 3 to 6, and the foresight azimuth from point 7 to point 3 was 220°, the backsight azimuth from point 3 to point 7 should be 40°. The north direction is now inside the interior angle.



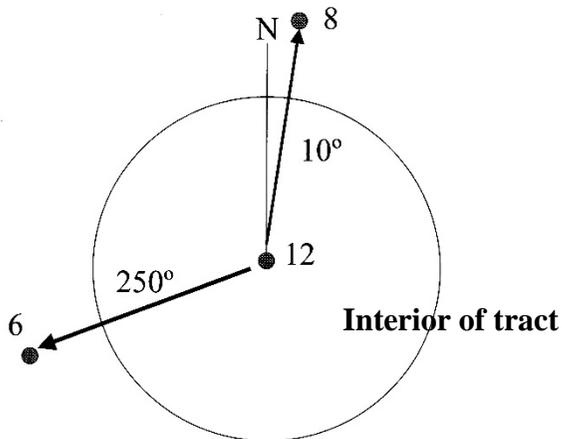
The interior angle about point 3 is then the sum of the angle between North and the projected backsight azimuth from point 3

to point 7 ( $40^\circ$ ), and between North and the projected foresight from point 3 to point 6 ( $50^\circ$ ). The deflection angle is  $90^\circ$ , since the total of the interior and deflection angles about a station (point) should sum to  $180^\circ$ .



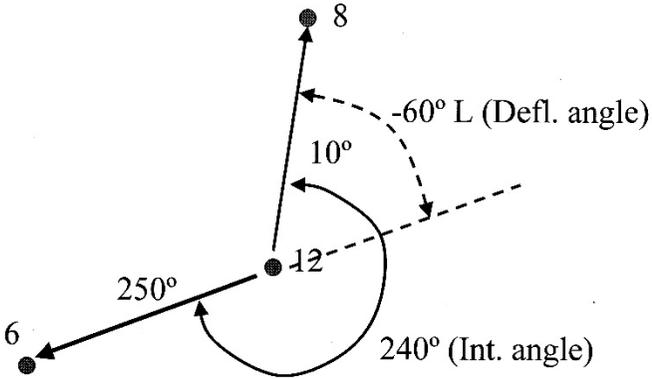
CASE 3 - The interior angle is greater than  $180^\circ$ .

If the direction of travel in the example below was from point 6 to 12 to 8, and the foresight azimuth from point 6 to point 12 was  $70^\circ$ , the backsight azimuth from point 12 to point 6 should be  $250^\circ$ . Here, assume that the interior of the tract is on the right side of these mapped measurements, thus it is greater than  $180^\circ$ .



The interior angle about point 12 is then the angle between the

250° backsight and the 10° foresight, or 240°. The deflection angle is  $-60^\circ$ , since the total of the interior and deflection angles about a station (point) should sum to  $180^\circ$ .



If North is "involved" the calculation of the interior angle should keep in mind the advice provided in Case 2.

### Important points

1. The sum of the interior angles should be  $(n - 2) (180^\circ)$ .
2. The sum of the deflection angles should be  $360^\circ$ .
3. The sum of the interior and deflection angles at a single station (point) should be  $180^\circ$ .

### 3.11 BALANCING LATITUDES AND DEPARTURES

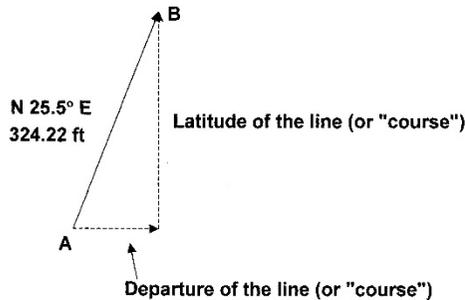
A surveyed transect, regardless of direction and length, can be described by latitudes and departures.

*Latitude* is the North-South component of a course. It describes how far (distance) and in what direction north or south (north is positive, south is negative) the transect is directed from point A to point B.

Latitude =  $\cos$  (foresight **bearing**) (distance)

*Departure* is the East-West component of a course. It describes how far (distance) and in what direction (east is positive, west is negative) the transect is directed from point A to point B.

Departure =  $\sin$  (foresight **bearing**) (distance)



In this case,

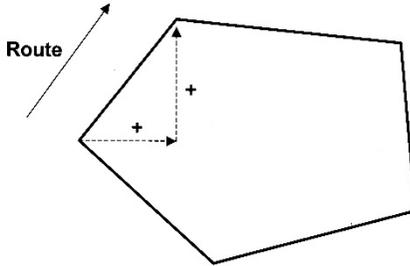
$$\begin{aligned} \text{Latitude} &= \cos (\text{foresight bearing}) (\text{distance}) \\ &= \cos (25.5^\circ) (324.22 \text{ feet}) \\ &= 292.6 \text{ feet} \end{aligned}$$

$$\begin{aligned} \text{Departure} &= \sin (\text{foresight bearing}) (\text{distance}) \\ &= \sin (25.5^\circ) (324.22) \\ &= 139.6 \text{ feet} \end{aligned}$$

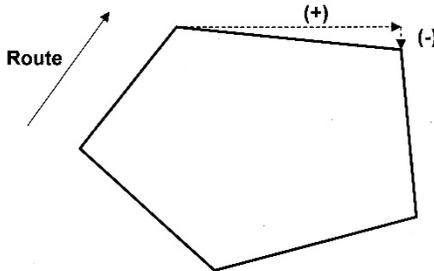
To interpret these results, one might imagine that the transect (course) has moved 292.6 feet to the north, and 139.6 feet to the east from point A to point B.

Latitudes and departures can be either positive or negative.

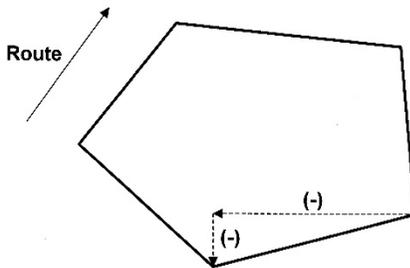
Transect moves north (positive latitude) and east (positive departure)



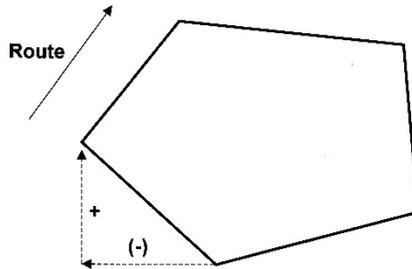
Transect moves south (negative latitude) and east (positive departure)



Transect moves south (negative latitude) and west (negative departure)

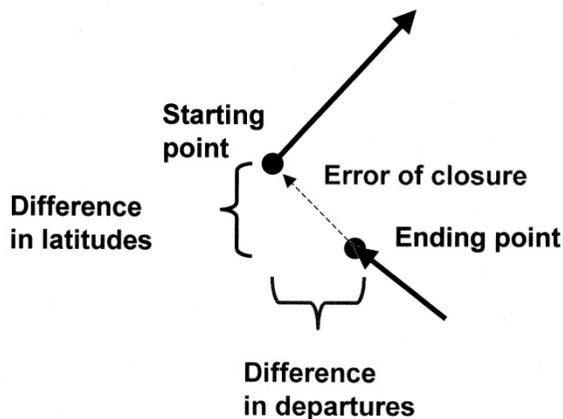


Transect moves north (positive latitude) and west (negative departure)



Important points

1. The sum of the latitudes around a closed traverse should equal zero (0).
2. The sum of the departures around a closed traverse should equal zero (0).
3. If the latitudes and departures do not sum to zero, there is an error of closure in the traverse, and the latitudes and departures need to be corrected.



The error of closure can be determined using the Pythagorean

Theorem:

$$a^2 + b^2 = c^2$$

or

$$(\text{sum of latitudes})^2 + (\text{sum of departures})^2 = \text{error of closure}^2$$

Based on the error of closure and the total length of the perimeter of the tract, the precision of the survey is expressed as

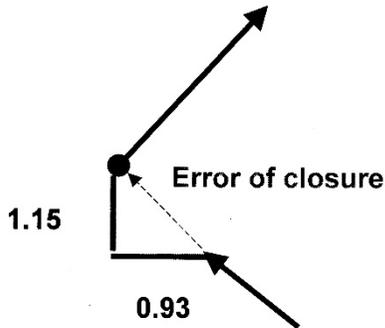
$$\text{Precision} = \left( \frac{1}{\left( \frac{\text{Total length of perimeter}}{\text{Error of closure}} \right)} \right)$$

Example Assume the following:

Sum of latitudes = 1.15 feet

Sum of departures = 0.93 feet

Perimeter of the traverse = 1,439 feet



$$\text{error of closure}^2 = (1.15 \text{ feet})^2 + (0.93 \text{ feet})^2$$

$$\text{error of closure}^2 = 2.1874 \text{ feet}$$

$$\text{error of closure} = 1.479 \text{ feet}$$

The precision is then

$$\text{Precision} = \left( \frac{1}{\left( \frac{1,439 \text{ feet}}{1.479 \text{ feet}} \right)} \right)$$

or more commonly state as 1 / 973. Therefore, for every 973 feet surveyed, the measurements are 1 foot off.

### Balancing latitudes

To correct, or balance, the latitudes associated with each side of a transect, you must first compute the "correction value" for each side of the transect.

$$\text{correction value} = \left| \text{sum of latitudes} \right| \left( \frac{\text{Length of side}}{\text{Perimeter of tract}} \right)$$

Then one of two cases are applied:

a) If the sum of the latitudes for the entire tract is *positive*, you need to subtract the correction value from each latitude.

$$\text{Each balanced latitude} = (\text{old latitude}) - (\text{correction value})$$

b) If the sum of the latitudes for the entire tract is *negative*, you need to add the correction value to each latitude.

$$\text{Each balanced latitude} = (\text{old latitude}) + (\text{correction value})$$

### Balancing departures

To correct, or balance, the departures associated with each side of a transect, you must first compute the "correction value" for each side of the transect.

$$\text{correction value} = \left| \text{sum of departures} \right| \left( \frac{\text{Length of side}}{\text{Perimeter of tract}} \right)$$

Then one of two cases are applied:

a) If the sum of the departures for the entire tract is *positive*, you need to subtract the correction value from each departure.

$$\text{Each balanced departure} = (\text{old departure}) - (\text{correction value})$$

b) If the sum of the departures the entire tract is *negative*, you need to add the correction value to each departure.

$$\text{Each balanced departure} = (\text{old departure}) + (\text{correction value})$$

### 3.12 MAKING A MAP FROM FIELD MEASUREMENTS

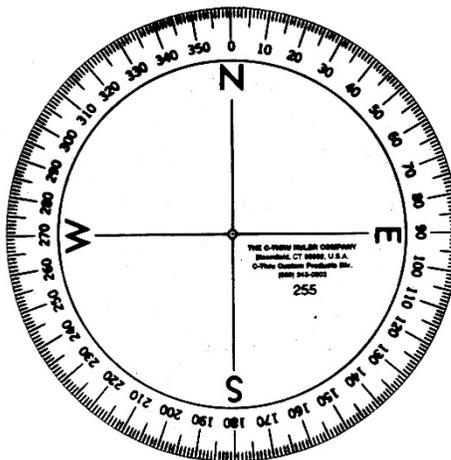
Map orientation and scale are two important assumptions that must be made prior to drawing a map from field measurements.

Scale was described earlier in Section 3.1. The two numeric scales were the equivalence scale and the representative fraction scale. One way to think about scale is to say to yourself "1 unit of X on the map represents Y units on the ground."

Map orientation is generally described by a north arrow. Generally, maps are designed with north facing upwards on a map. However, if the shape of a mapped tract will not permit it to fit nicely on the paper you are using, north can point in any other direction as long as north is described correctly.

#### Drawing a map with field data and a protractor

Any type of protractor can be used to draw a map by hand. The following instructions use a 360° protractor (shown below), but they can be extended to 180° protractors.



Assume that you surveyed a 3-sided tract in the field, and obtained the following measurements:

Beginning point A (travelling to point B)

Distance = 300 feet

Azimuth =  $25^\circ$

Point B (travelling to point C)

Distance = 224 feet

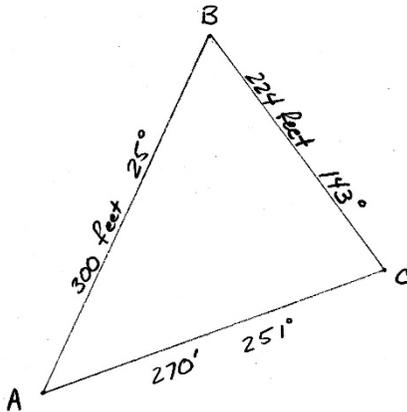
Azimuth =  $143^\circ$

Point C (travelling back to beginning point A)

Distance = 270 feet

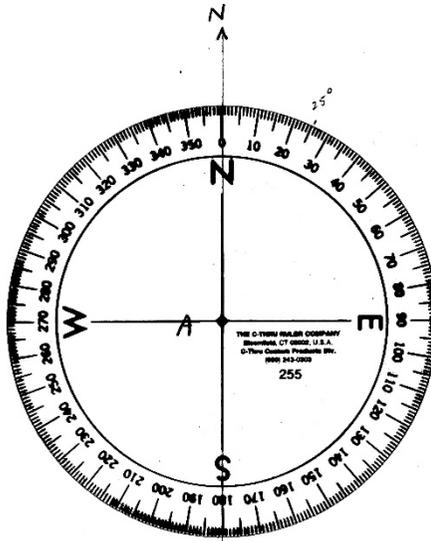
Azimuth =  $251^\circ$

Assume also that you want to draw a map at a scale of  $1'' = 100$  feet. You would first need to visualize a rough sketch of the area to determine where, on a blank piece of paper, you would begin drawing the map from point A. In the rough sketch below, you can see that point A is on the left-hand side of the tract (with north facing upward). Therefore, you need to pick a point on the left-hand side of a blank piece of paper to begin drawing the map.



Follow the steps outline below to draw a map to scale on a blank piece of paper.

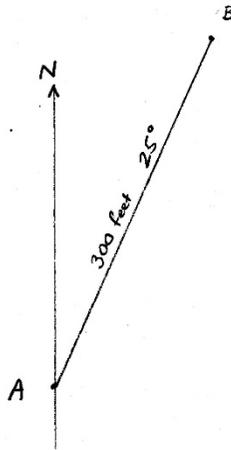
1. Choose the starting point.
2. Draw a meridian (north-south line) through the starting point.
3. Lay the protractor on the paper with the north-south lines aligned with the meridian. The center (or index) of the protractor must be placed directly on top of point A.
4. Place a dot, or "tick" mark at the appropriate azimuth from point A to point B (in this case, 25°).



5. Scale out the distance from point A to point B. In our case, it is 300 feet (horizontal ground distance) from point A to point B, which equals 3 inches on the map when using a scale of 1" = 100 feet. To determine the map distance, the formula below was used:

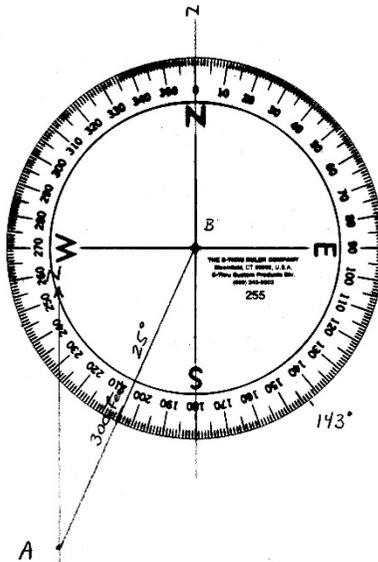
$$\text{map distance} = 300 \text{ feet} \left( \frac{1 \text{ inch}}{100 \text{ feet}} \right) = 3 \text{ inches}$$

6. Draw the map distance (3 inches) from point A toward the tick mark representing the direction to point B.



7. Repeat steps 2-6 for the edge of the tract that is represented by side B-C.

First draw a new meridian through point B. Then locate the azimuth from point B to point C ( $143^\circ$ ).

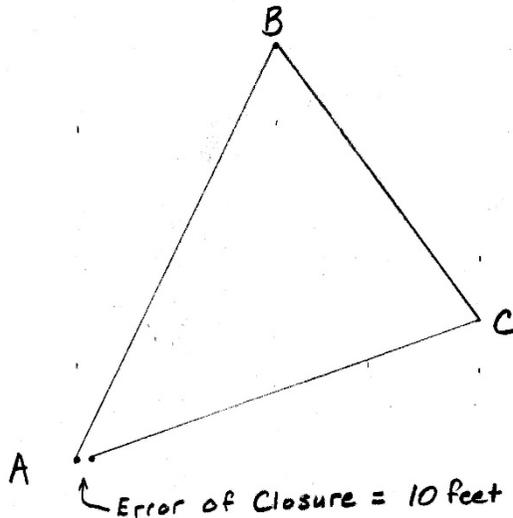


Then draw a line representing the appropriate distance from point B to point C.

In this case, the map distance is 2.24 inches. To precisely draw this on a map, you could use the 50 scale of the Engineer's scale. Here, there are 50 parts per inch, therefore, when developing a map where each inch represents 100 feet, each tick mark on the 50 scale represents 2 feet. Thus when using the 50 scale, you would need to locate the place on the scale where 112 tick marks resides.

8. Repeat steps 2-6 for any remaining sides of the tract.

When you are done, you might find that the traverse does not "close." this is the *error of closure*, and it should be measured and estimated in terms of ground distance. In the case we have been following the error of closure is 10 feet.



When taken in context of the entire survey, which has a length of 794 feet, the precision of the survey can be stated as

$$precision = \left( \frac{\text{length of survey}}{\text{error of closure}} \right)$$

$$precision = \left( \frac{794 \text{ feet}}{10 \text{ feet}} \right) = 79.4$$

This can then be put into words: The survey precision is 1 / 79.4, or for every 79.4 feet surveyed, we were 1 foot off.

### Important points

1. The accuracy of hand-drawn maps will increase with experience, skill, and diligence of the map maker.
2. Errors of closure can be corrected using methods described previously.

### Using balanced latitudes and departures

If you have the balanced latitudes and departures, you should be able to draw a map that has no error of closure.

After establishing the first point (corner) on a map, and giving it some arbitrary X, Y coordinate, each subsequent point (corner) has a coordinate can be defined.

Keep in mind that X = east-west coordinate, and Y = north-south coordinate.

subsequent point X = previous point X + balanced departure

subsequent point Y = previous point Y + balanced latitude

### 3.13 ESTIMATING THE SIZE OF A TRACT - DOT GRIDS

One of the most common ways for natural resource managers to estimate areas of land from a map or aerial photograph is with the assistance of a dot grid.

A dot grid is a sheet of transparent plastic that includes a number of small dots, sometimes 64 per square inch, sometimes 100 per square inch, sometimes other densities. What one would do with a dot grid is place it on top of a drawn map or aerial photograph, and count the number of dots within an area of interest. This number leads to an estimate of square inches of mapped area, which then can lead to an estimate of actual ground area (acres). To be able to use a dot grid to estimate areas requires the following information:

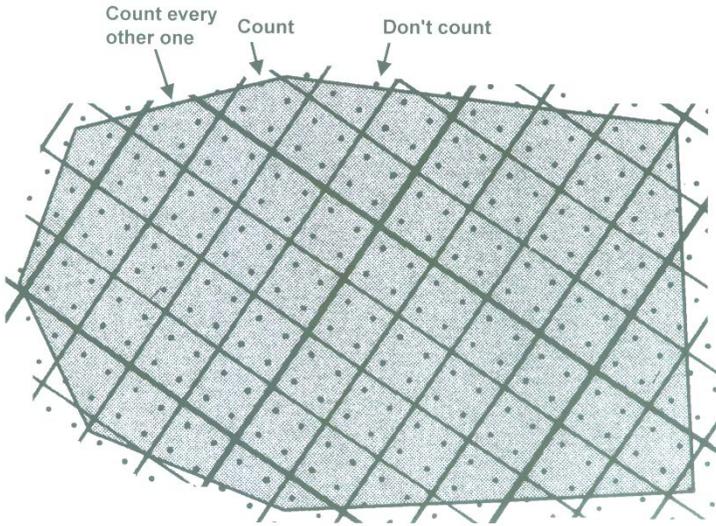
- The density of the dot grid
- The number of dots within an area of interest
- The scale of the map or aerial photograph

To develop an estimate of acreage with a dot grid, follow the steps outlined below:

1. Randomly (not purposefully) place the dot grid on your map.
2. Count the dots inside of your area of interest.

*Caveats:*

- a. If a dot lies directly on the boundary of your area of interest, either count every other one of these, or count them as 1/2 dots (either system can be used, but only use one of the two systems).
- b. If a dot touches the boundary of your area of interest, but lies most inside the area of interest, count the dot.
- c. If a dot touches the boundary of your area of interest, but lies mostly outside the area of interest, do not count it.



3. Repeat steps 1 and 2 at least two more times to get at least three estimates of the dot count within your area of interest.
4. Average the dot counts from your 3+ dot count readings.
5. Divide the average dot count by the density of the dot grid to arrive at the mapped size of the area of interest. For example, if the average dot count was 312 dots, and the density of the dot grid was 64 dots per inch, the mapped size of the area of interest is 4.875 square inches.
6. Determine how many acres each square inch of the map represents.

For maps that use a representative fraction scale, the following equation can be used:

$$\text{Each square inch represents } \left( \frac{\text{scale}^2}{(144)(43,560)} \right) \text{ acres}$$

In the equation above the "scale" is the representative fraction

scale (e.g., for a 1:24,000 map, it is 24,000; for a 1:15,840 map it is 15,840). The value "144" converts the scale<sup>2</sup> to square feet. The value "43,560" converts the square feet to acres.

**Example** A map has a representative fraction scale of 1:15,840. This implies that each inch *on the map* represents 15,840 *on the ground*. Therefore, each square inch on the map represents 40 acres on the ground.

$$\text{Each square inch represents } \left( \frac{15,840^2}{(144)(43,560)} \right) = 40 \text{ acres}$$

For maps that use an equivalence scale (e.g., 1" = 10 chains), you need to think about the process a little differently. For example if one inch on the map equaled 10 chains on the ground, then one square inch on the map equals 10<sup>2</sup>, or 100 square chains on the ground, or 10 acres.

7. Multiply the square inches measured (from step 5) and the area each square inch represents (from step 6) to arrive at an estimate of the size of the area of interest.

**Example** The scale of your map is 1:24,000. Your dot grid has a density of 64 dots per square inch. You counted dots three times (steps 1-3 above), and arrived at the following dot counts within your area of interest: 173, 176, 172. What is the actual size, in acres, of your area of interest?

Using the guidance from step 4 above, the average dot count is 173.67 dots within your area of interest.

Using the guidance from step 5 above, the square inches represented by your area of interest on the map is 2.714.

$$\left( \frac{173.67 \text{ dots}}{64 \text{ dots per square inch}} \right) = 2.714 \text{ square inches}$$

Using the guidance from step 6 above, you found that each square inch on the map represents 91.83 acres on the ground.

$$\left( \frac{24,000^2}{(144)(43,560)} \right) = 91.83 \text{ acres per square inch}$$

Using the guidance from step 7 above, you determine that the size of your area of interest is 249.2 acres.

$$(2.714 \text{ in}^2) (91.83 \text{ acres per in}^2) = 249.2 \text{ acres}$$

### Important points

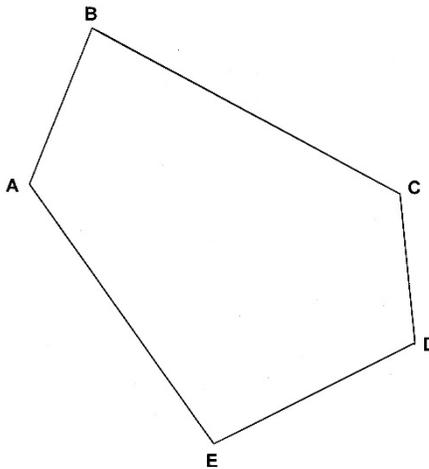
1. The transparent plastic dot grid should not be moved as you are counting dots. Once it is laid down on top of a map or aerial photograph, hold it firmly in place or tape it down.
2. If you have access to a wax pencil, dots can be marked as they are counted.
3. If three dot counts result in distinctly different numbers (e.g., 120, 139, 117), more counts are needed, or perhaps even better, you should start over.
4. To make the count more reliable, dots should be counted in rows or columns when the tract is irregular.

### 3.14 ESTIMATING THE SIZE OF A TRACT - SCALING TRIANGLES

Another way to develop an estimate of the size of a mapped area of interest is to subdivide the area of interest into triangles, scale them out, and determine the size of each triangle with the formula

$$\text{size (area)} = \left(\frac{1}{2}\right)(\text{base})(\text{height})$$

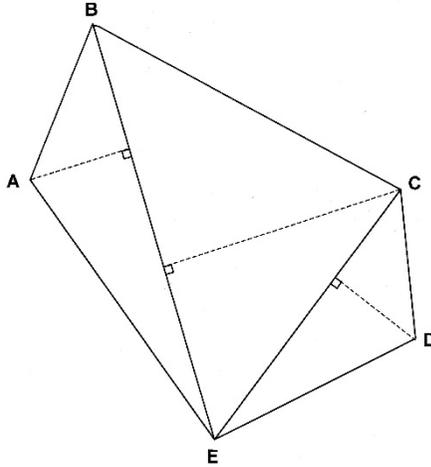
For example, assume you have surveyed the following 5-sided polygon in the field:



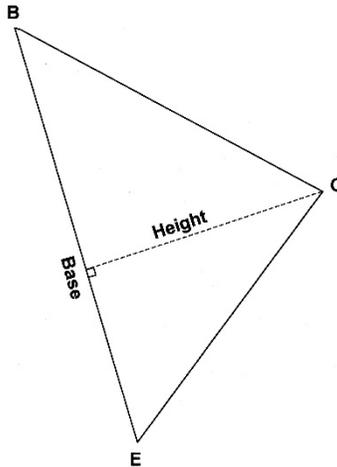
The polygon can be subdivided into three triangles, ABE, BCE, and CDE.

A temporary line can be extended from one corner of a triangle to an opposite side to form two right triangles.

This extended, temporary line can be considered the height of the triangle. The side to which the line was extended can be considered the base of the triangle.



In the case of triangle BCE, the base is the side B-E.



You can measure the distance of the height and the base of each triangle on the map, and knowing the scale of the map, you can estimate the length of each. Using an engineer's scale, we might find that for triangle BCE (from the original map):

Base = 4.60 inches

Height = 2.66 inches

**Example** Using the 5-sided polygon described above, assume the following:

Triangle ABE

Base = 4.60 inches

Height = 1.10 inches

Triangle BCE

Base = 4.60 inches (same base as triangle ABE)

Height = 2.66 inches

Triangle CDE

Base = 3.32 inches

Height = 1.18 inches

Assume also that the scale of the map was 1" = 200 feet. What is the length of the base and the height of each triangle (in feet), the size of each triangle (in acres), and the size of the polygon (in acres)?

Triangle ABE

Length of base = (4.60 in.) (200 feet / in.) = 920 feet

Length of height = (1.10 in.) (200 feet / in.) = 220 feet

Triangle BCE

Length of base = (4.60 in.) (200 feet / in.) = 920 feet

Length of height = (2.66 in.) (200 feet / in.) = 532 feet

Triangle CDE

Length of base = (3.32 in.) (200 feet / in.) = 664 feet

Length of height = (1.18 in.) (200 feet / in.) = 236 feet

Size of triangle ABE:

$$\left(\frac{1}{2}\right)(920 \text{ feet})(220 \text{ feet}) = 101,200 \text{ ft}^2 = 2.32 \text{ acres}$$

Size of triangle ABE:

$$\left(\frac{1}{2}\right)(920 \text{ feet})(532 \text{ feet}) = 244,720 \text{ ft}^2 = 5.62 \text{ acres}$$

Size of triangle ABE:

$$\left(\frac{1}{2}\right)(664 \text{ feet})(236 \text{ feet}) = 78,352 \text{ ft}^2 = 1.80 \text{ acres}$$

Size of the polygon:

$$2.32 \text{ acres} + 5.62 \text{ acres} + 1.80 \text{ acres} = 9.74 \text{ acres}$$

### Important points

1. This process for determining the area of a piece of land assumes that the mapped area of interest can be subdivided into triangles, which may not be possible for irregularly-shaped parcels, or parcels with more than 5 or 6 sides.
2. This process assumes that the piece of land was drawn correctly using the azimuths and distances provided, and was drawn to the correct scale.
3. This process assumes that the line that extends from one corner to an opposite side was drawn at a right angle to the opposite side.
4. This process assumes that the base and the height of each triangle were scaled correctly using an engineer's scale. The finer scales (40, 50, 60) may be needed to arrive at more accurate measurements of length than the 10 or 20 scales.

### 3.15 ESTIMATING THE SIZE OF A TRACT - DOUBLE MERIDIAN DISTANCES

The area of a closed traverse can be determined using what are called "double meridian distances." Here, balanced departures are needed (see Section 3.11) from a traverse.

Assume that you have the following balanced departures from a 4-sided traverse:

Course	Balanced latitudes	Balanced departures
Point A to Point B	+ 283.36	+ 135.08
Point B to Point C	- 176.00	+ 292.24
Point C to Point D	- 377.26	- 319.36
Point D to Point A	+ 269.90	- 107.96

(Notice that the sum of the balanced latitudes is 0 and the sum of the balanced departures is 0)

To calculate the double meridian distances (DMD), two rules of thumb are used:

- a) The first DMD is simply the balanced departure of the first side (course) of the traverse.
- b) Each subsequent DMD uses the following formula:

$$\begin{aligned} \text{DMD} &= (\text{previous DMD}) \\ &+ (\text{balanced departure of the previous side or course}) \\ &+ (\text{balanced departure of the current side or course}) \end{aligned}$$

Therefore, the DMD associated with each side of the transect becomes:

Course	Balanced departures	DMD
Point A to Point B	+ 135.08	135.08
Point B to Point C	+ 292.24	562.40
Point C to Point D	- 319.36	535.28
Point D to Point A	- 107.96	107.96

The DMD for Point B to Point C = (135.08 + 135.08 + 292.24)

The DMD for Point C to Point D = (562.40 + 292.24 - 319.36)

The DMD for Point D to Point A = (535.28 - 319.36 - 107.96)

The "double area" for each side (course) of the transect needs to next be computed.

$$\text{Double area} = (\text{DMD}) (\text{balanced latitude})$$

Course	DMD	Double area
Point A to Point B	135.08	38,276
Point B to Point C	562.40	- 98,982
Point C to Point D	535.28	- 201,940
Point D to Point A	107.96	29,138

The double area for Point A to Point B =  $(135.08) \times (283.36)$

The double area for Point B to Point C =  $(562.40) \times (-176.00)$

The double area for Point C to Point D =  $(535.28) \times (-377.26)$

The double area for Point D to Point A =  $(107.96) \times (269.90)$

The sum of the double areas is then -233,508.

Using the sum of the double areas, and ignoring the sign, the next step is to divide the double area by 2.

$$\left( \frac{| \text{double area} |}{2} \right)$$

$$\left( \frac{| -233,508 |}{2} \right) = 116,754$$

The units of the result of this calculation are square feet.

Then, you would divide this value by the number of square feet in an acre (43,560) to arrive at acres.

$$\text{Area} = \left( \frac{116,754}{43,560} \right) = 2.68 \text{ acres}$$

Performing these calculations by hand is moderately difficult. Using a spreadsheet makes the effort less difficult and time consuming.

### 3.16 FINAL THOUGHTS

Errors in field measurements can be categorized as:

1. Mistakes. These are incorrect measurements, for one reason or another. They are avoidable and detectable.

Causes: Carelessness, ignorance

2. Systematic errors. These are errors of the same size and magnitude with each subsequent measurement. They are avoidable yet may be hard to detect.

Same sign (+ or - correct measurement)

Same size

Causes: Devices have problems (kinks in tapes, broken and repaired tapes, or bent compass needles), fatigue (pace gets shorter when you are tired).

3. Accidental errors. They arise due to limitations of the measuring devices, limitations of humans to read the measurement devices. They are as likely to be positive as they are to be negative in relation to the correct measurement. They are unavoidable and may be hard to detect.

Causes: Devices that have measurement intervals that cannot provide precise readings, or are difficult for humans to interpret precise readings.

**Example** Your compass reports azimuth angles in  $2^\circ$  intervals. Therefore, you may either occasionally forget this and assume they are  $1^\circ$  intervals, or you may read the intervals incorrectly.

### 3.17 REFERENCES

- National Geophysical Data Center. 2015. Magnetic declination. NOAA National Geophysical Data Center, Boulder, CO. <https://www.ngdc.noaa.gov/geomag/calculators/magcalc.shtml#declination> (accessed 11/21/19).
- U.S. Centennial of Flight Commission. 2000. The Poles. U.S. Centennial of Flight Commission, Washington, D.C. <http://centennialofflight.net/essay/Dictionary/Poles/DI81.htm> (accessed 11/21/19).

## 4.0 GLOBAL POSITIONING SYSTEMS

Global positioning systems (GPS) are an important tool for natural resource managers. Wildlife biologists use GPS to document the location of nests and other critical habitat. Foresters use GPS to delineate treatment areas and develop standard geographic information systems (GIS) databases for roads and other management-related infrastructure. Fisheries biologists and hydrologists use GPS to map stream systems. Recreation resource managers use GPS to map trail systems. A variety of professionals use GPS in conjunction with wildfire monitoring and suppression activities. All natural resource managers along with the general public use GPS to navigate.

In 1983, the United States made the decision to provide GPS service free of charge, and openly, to the world.

The U.S. GPS system was initially funded by the Department of Defense, and the official name is NAVSTAR GPS. The system is currently managed by the U.S. Air Force, 50<sup>th</sup> Space Wing, at Schriever Air Force Base in Colorado Springs, Colorado. The cost of maintaining the NAVSTAR GPS program is estimated to be over \$1.2 billion per year.

After completing this Chapter, you will be able to:

1. Understand the elements of GPS and how they interact to allow you to navigate and map landscape features.
2. Understand the different types of GPS receivers available.
3. Understand error sources and calculate the error in GPS measurements.

## 4.1 CONTROL STATIONS, BASE STATIONS, AND DIFFERENTIAL CORRECTION OF GPS DATA

While most people think of GPS in terms of the receiver that they use, GPS is actually a system of satellites, receivers, control stations, base stations, and software for navigation and mapping the earth's features.

We will discuss the satellites and receivers in various parts of this Chapter.

*Control stations* are ground-based facilities used to monitor and control the movement of the satellites. They are operated by the U.S. government. There are five passive monitoring control stations across the globe. There is a single master control station in Colorado Springs. Navigation and ephemeris data are determined from the monitored signals and are uploaded to each satellite once or twice per day.

*Base stations* are GPS receivers that are generally not associated with the control stations, and are installed at a location that has been surveyed to a high degree of accuracy. Therefore the position that it calculates given the signals it receives can be compared to "truth" (the surveyed position) and a measure of error (distance and direction) for each moment in time can be estimated. The data from your GPS receiver are then compared to this error and adjustments are made. Since most GPS errors affect data (rover and base station) nearly equally, some error can be cancelled out in the calculations.

Base stations are GPS receivers that are located in open areas (on top of buildings, for example), collect data 24 hours a day (ideally), and provide these data to users of GPS. Most base station data can either be obtained over the Internet or through a file transfer protocol (ftp) connection. The network of Continuously Operating Reference Stations (CORS) can be accessed over the Internet

([www.ngs.noaa.gov/CORS/data.shtml](http://www.ngs.noaa.gov/CORS/data.shtml)). These provide base station data for free to any user of GPS.

As long as the data you have collected is located within about 200 miles of a base station, the base station data can be reasonably used to further enhance the accuracy of your work.

*Differential Correction* is the process of correcting data collected by a roving GPS receiver using data collected by a stationary GPS receiver positioned at a fixed location (i.e., a base station). To differentially correct GPS data, you need to download your GPS data to a computer, acquire the appropriate base station data, and use software to make the corrections. Instructions for doing this with a Trimble GeoExplorer 3 are located in Section 4.17.

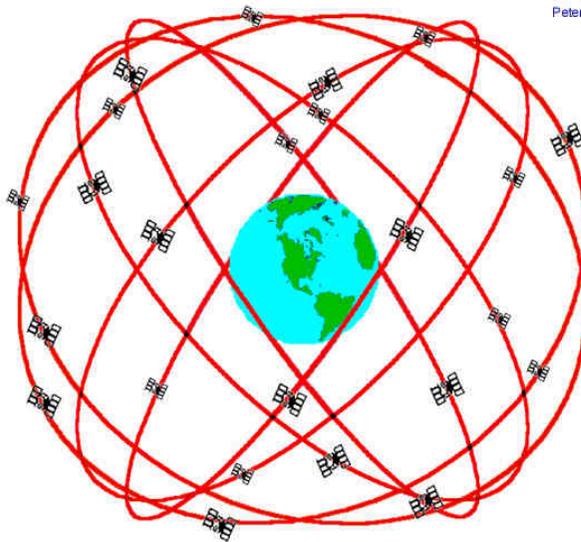


A base station. (U.S. Geological Survey 2013)

## 4.2 GPS SATELLITE SYSTEM

NAVSTAR GPS satellites orbit the earth at an elevation of about 12,900 miles. These satellites orbit the earth every 12 hours - they are not geostationary (they do not remain in one place above the earth).

Given the rotation of the earth, satellites repeat the exact ground track once every day. These ground tracks are "orbital planes," of which there are 6, and each contains at least 4 satellites. This arrangement ensures that at least 6 satellites are within a line of sight from almost everywhere on earth, all the time.



**GPS Nominal Constellation**  
**24 Satellites in 6 Orbital Planes**  
**4 Satellites in each Plane**  
**20,200 km Altitudes, 55 Degree Inclination**

As of July 2014, there were 31 satellites in the system. They are given a number by the order in which they were launched, or by their pseudo-random noise (PRN) code number, which is a unique integer used to code the signal from that satellite.

### 4.3 GPS SATELLITES

Because GPS satellites are also called "space vehicles," within some GPS receivers you may see the acronym "SV."



Los Angeles Air Force Base (2014)

Each satellite transmits signals via a form of radio wave to the earth, each signal contains the satellite's orbital elements, clock behavior, system time, and status. Satellites also transmit an almanac, which provides approximate data for each of the other active satellites in the system. The almanac helps the receiver to find all of the other satellites once the first satellite signal has been acquired.

The three basic pieces of information that a satellite transmits are:

1. An ephemeris - its own location
2. An almanac - the approximate location of other satellites
3. A time signal - to enable triangulation of a position

## 4.4 GPS FREQUENCIES

Two basic frequencies of signals are transmitted by GPS satellites, the L1 and L2, both of which are located in the L-band of the electromagnetic spectrum, between microwaves and radio waves:

L1	1575.42 MHz
L2	1227.6 MHz

*Coarse / acquisition code (C/A)* can be obtained from the L1 frequency. This is the most widely used GPS signal, yet the least accurate of signals. It is simple and cost-effective to use, thus most of the GPS receivers used in natural resource management are designed to capture and utilize this code.

*Carrier Phase code* can be obtained from the L1 frequency as well. This facilitates more accurate position determination than the C/A code, but requires more time to use, and thus is more expensive to use. Surveying and benchmarking operations, using sophisticated GPS receivers, capitalize on the use of this code.

*P-Code* is available on both the L1 and L2 frequencies, and using it can result in more accurate position determination than the Carrier Phase code. However, the P-code is reserved for military use.

These waves of electromagnetic energy can pass through some forest canopies, but may be blocked by tree trunks or other landscape features, particularly if the signal arrives at the receiver from a low angle in the atmosphere's horizon.

The speed of each signal is 186,000 miles per second. If a satellite is 12,900 miles above the earth, the time it takes for a signal to leave the satellite and reach earth is about 1/14 of a second.

## 4.5 SATELLITE CLOCKS

GPS satellites are equipped with four atomic clocks, which provide highly precise measurements of time. Instead of clocks that use pendulums or springs, atomic clocks rely on the natural resonance of atoms (cesium or rubidium) that provide a steady "tick."

Each chemical element absorbs and emits electromagnetic energy at its own resonance, or frequency, and scientists have found that these resonances are stable over time.

One unit of time, the second, has been precisely defined as (Breakiron 2003):

*9,192,631,770 cycles of microwave light absorbed or emitted by the hyperfine transition of cesium-133 atoms in their ground state undisturbed by external fields*

Although incredibly precise, atomic clocks are not perfect, given certain instabilities in the orbiting satellites. Thus they are frequently adjusted to the clocks that are used in the ground control stations.

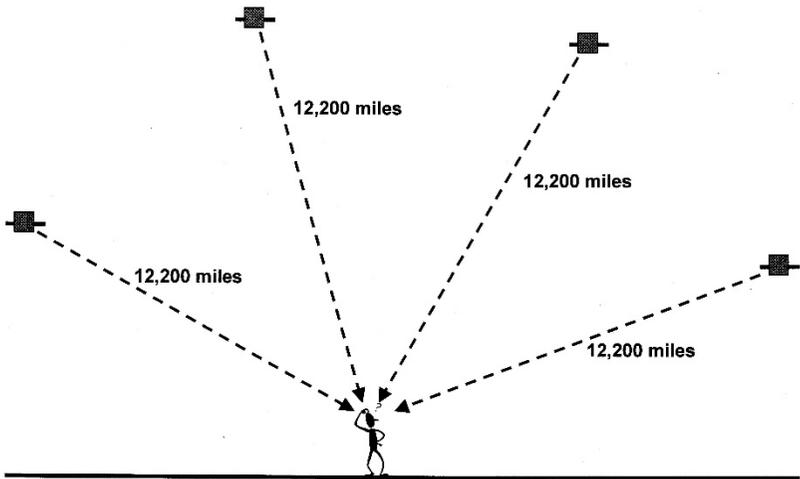
A slight variation in clock time, given the very fast speed of signal transmission and reception, could result in inaccurate position determination. Therefore, the clock within a GPS receiver must be synchronized exactly with the satellite's clock.

Stability is relative, however, and it has been suggested that it would take almost 32,000 years for a GPS atomic clock lose one second of time (Phillip and Voss 2002).

## 4.6 How GPS Works

Position determination and navigation rely on triangulation of GPS signals. To understand your horizontal position on the earth requires three signals. A GPS receiver will calculate how long it took for each signal to be received, and in conjunction with the almanac (which provides an approximation of where each satellite is located in the sky), can determine a distance to each satellite. Three distances and their associated direction can provide enough information to triangulate a horizontal position on the earth.

A fourth satellite signal is required to understand your elevation, or vertical position, on the earth.



## 4.7 SOME IMPORTANT GPS TERMINOLOGY

*PRN* (pseudo-random noise) - A unique integer assigned to each satellite which is used to code the signal from that satellite. Sometimes a satellite is referred to by this value.

*SNR* (signal-to-noise ratio) - A measure of the information content of a GPS signal relative to the signal's noise. As this proportion decreases, information gets lost in the noise. A value above 20 is very good. The quality of a position is degraded if the signal strength of any satellite in the constellation is below 6. The default SNR mask is 6.

*SNR Mask* - The lower limit on SNR, beyond which no data will be collected. Can be modified by the user of a GPS receiver.

*DOP* (dilution of precision) - A term used to describe the strength of a satellite configuration on the accuracy of data collected with GPS receivers. When satellites are bunched together in the same area of the sky view, the satellite geometry is termed "weak." The DOP will therefore be high in this case, and low when the satellites are spread apart in the sky. Satellites that cannot be acquired due to landscape feature obstructions will weaken and increase the DOP.

*VDOP* (vertical dilution of precision). Relates to the elevation suggested by GPS data.

*HDOP* (horizontal dilution of precision). Relates to the horizontal position suggested by GPS data.

*PDOP* (positional dilution of precision) - Most widely presented value in GPS receivers. This is the "mean" DOP value, and provides an overall rating of precision based on horizontal and vertical dilution of precision. The best PDOP value would be obtained with one satellite directly above the receiver and three others evenly spaced across the sky.

## PDOP

value      Comment on data quality

---

1	This PDOP provides the highest possible confidence in data accuracy, and is used for applications that demand the highest level of precision.
2-3	In this range, the positional measurements are accurate enough for all but the most sensitive applications.
4-6	This is the minimum level for mapping applications and for making business decisions.
7-8	Here, positional measurements could be used, but a better configuration of satellites is suggested.
9-20	Positional measurements with this PDOP provide low confidence in data quality.
21-50	Measurements using this PDOP range may be inaccurate by more than 150 feet.

---

*PDOP Mask* - The upper limit on PDOP, beyond which no data will be collected. Can be modified by the user of a GPS receiver.

*Elevation Mask* - A filter used to ignore satellites that are below a certain angle on the horizon.

*Offset* - A set of data (distance, angle, and inclination) describing a constant position from an unreachable feature. The collected GPS data will be adjusted using the offset to place the feature in its correct position.

## 4.8 SOURCES OF ERROR

*Atmospheric interference* - Distortion occurs when the GPS signals travel through a greater portion of the ionosphere and troposphere. The ionosphere is the upper atmosphere, where electromagnetic wavelengths in the radio frequency range can be distorted by a high density of free electrons. The troposphere is the lowest portion of the atmosphere, where most of the water vapor and aerosols reside. These too can distort radio signals. The least amount of interference occurs when satellites are directly overhead the GPS receiver. However, this also implies a high PDOP, and thus lower quality data.

*Ephemeris* - The ephemeris provides the estimated coordinates of the SV in the sky. If the SV is not quite where it was suggested to be, the accuracy of the position that is determined may be affected. An ephemeris is only valid for about 4 hours. An example of ephemeris data is shown below.

```
EPHEMERIS FOR SATELLITE 24 :
PRN number for data ..... 24
Issue of ephemeris data ..... 179
Semi-Major Axis (meters) ..... 2.65599E+07
C(ic) (rad) ..... -1.02445E-07
C(is) (rad) ..... -1.22935E-07
C(rc) (meters) ..... 168.656
C(rs) (meters) ..... -63.3125
C(uc) (rad) ..... -3.48687E-06
C(us) (rad) ..... 1.1526E-05
Mean motion difference (rad/sec) ..... 3.94802E-09
Eccentricity (dimensionless) ..... 0.00623617
Rate of inclination angle (rad/sec) .. 1.05004E-10
Inclination angle @ ref. time (rad) .. 0.976756
Mean Anomaly at reference time (rad) . 1.79689
Corrected Mean Motion (rad/sec) ..... 0.000145861
Computed Mean Motion (rad/sec) ..... 0.000145858
Argument of perigee (rad) ..... -2.06498
Rate of right ascension (rad/sec) .... -7.67032E-09
Right ascension @ ref time (rad) ..... -2.4059
Sqrt (1 - e^2) ..... 0.999981
Sqr root semi-major axis, (m^1/2) .... 5153.63
Reference time ephemeris (sec) ..... 252000
```

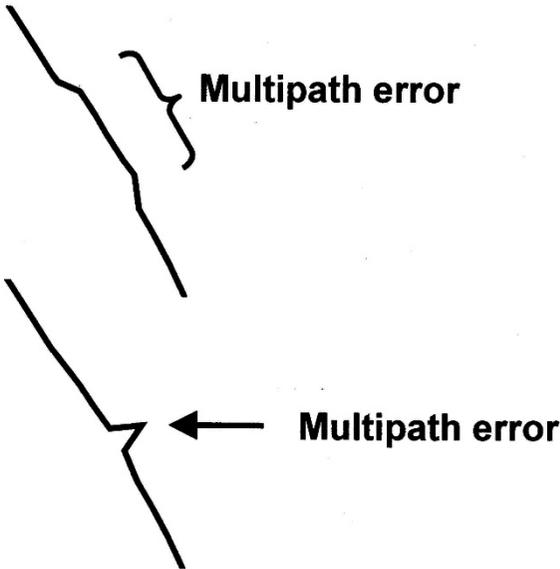
*Atomic clock* - Since the GPS signals travel at the speed of light, if the time kept atomic clocks is different from GPS time by even a microfraction of a second, the calculated ground position using that satellite can be affected. Atomic clocks, while very accurate, are still subject to minor deviations.

*GPS Receiver clock* - The time required for a signal to be transmitted from a satellite and received by a GPS receiver is a direct input into the determination of a position on the ground. Once a GPS receiver has the appropriate set of satellite ephemeris data (describing the position of each satellite), and once it understands GPS time, positions on the ground can be determined. A GPS receiver can not become precisely synchronized with every satellite, however, since GPS time is a composite of all monitoring station and satellite atomic clocks. Therefore, if for some reason the GPS receiver clock is not exactly synchronized with a satellite's atomic clock, error can occur in the determination of the distance from a satellite to the receiver, thus the calculated position will be somewhat inaccurate. This source of error is larger than the SV atomic clock error.

*Signal to noise ratio* - Satellite signals do not penetrate buildings, trees, or other similar objects. When collecting GPS data, consider placing the antenna a meter or two away from a tree.

*Multipath* - These errors occur when satellite signals bounce off of other landscape features, such as buildings, trees, people, or the ground, before entering the GPS receiver. Some GPS receivers have the technology to understand that certain signals from certain satellites are delayed, as compared to other signals it has already received from the satellite. The GPS receivers then ignore these delayed signals, assuming they are the product of multipath. Imagine that you were using a GPS receiver to map the location of a road or a stream (or other line feature), the following two cases represent typical multipath error that may be found in your data, even after differential correction has been

performed.



*Other Sources of Error* - A shadowing effect, which is a result of excess attenuation (gradual loss of intensity) of the direct path of a satellite signal to a GPS receiver, can occur when working in forested conditions. This occurs when GPS signals pass through vegetation. Shadowing of signals can be so severe that a GPS receiver is resorted to using only higher quality (un-attenuated) multipathed signals.

For all types of data, interfering with the antenna reception may be a problem. In these cases of "receiver error," holding a GPS receiver close to your body may result in the blockage of signals from some portion of the available set from the constellation. Further, if the direct path from a satellite to a GPS receiver is blocked, a multipathed version of the signal may unavoidably be used. Many seasoned users of GPS suggest using an external antenna that can be held several feet over one's head, in order to avoid errors such as these.

### Important points

1. Horizontal positions obtained from GPS are typically two to five times more accurate than vertical positions (elevations).
2. Multipath error represents the largest source of inaccuracy in GPS data collected for natural resource management purposes, particularly in areas with dense vegetation, large trees, or severe topographic relief.
3. A general error budget for GPS data collected in forested conditions is shown below. These values are rough, admittedly, and are open to debate. They are formed by our general impression of what we have noted through field studies, and by what we have read in the literature.

Source of error	Percent
Multipath	30-50%
Atmospheric / Ionospheric	25-30%
Receiver error	10-25%
Satellite clock error	5-10%
Ephemeris error	5-10%

## 4.9 WAAS AND DGPS

The *wide-area augmentation system* (WAAS) was developed by the U.S. Federal Aviation Administration to provide additional accuracy for the airline industry and other uses of flying machines. A network of ground stations are distributed throughout North America to monitor and measure GPS signals, and to send real-time correction messages to two geostationary satellites positioned over the equator. These corrected signals are then transmitted to WAAS-enabled GPS receivers to instantaneously correct their GPS positions.

*Differential GPS* (DGPS) is a service that was developed by the U.S. Coast Guard in 1999. This system includes two control centers and about 80 remote ground-based broadcast beacons. The system provides service for coastal regions of the U.S., and allow real-time differential correction of GPS data. *Static mode DGPS* is where the rover and the base station remain in fixed places (are stationary). *Kinematic mode DGPS* is where the base station remains in a fixed position, yet the rover moves from unknown location to unknown location.

Both systems require dual frequency capability in GPS receivers (one frequency is a signal from the GPS satellites, the other frequency is the signal from the WAAS or DGPS satellites or beacons).

### Important points

1. The WAAS service is only available in North America.
2. Since only two satellites transmit the WAAS signal, reception may be an issue in the field.
3. Differential correction can improve WAAS or DGPS data.
4. Obtaining a real-time "lock" may take some time.

## 4.10 ACCURACY ASSESSMENTS

Accuracy of data collected with GPS receivers can be assessed using both Root Mean Square Error (RMSE) and 50% Circular Error Probable (CEP<sub>50</sub>). CEP<sub>50</sub> refers to the radius of a circle that half of the sample points fall into.

RMSE is calculated using the example shown below, and represents the average error of position locations after squaring each individual position, summing them, averaging them, then taking the square root of the average. By squaring the error, larger errors have more of an influence on the data quality assessment than smaller errors.

Test point	Northing difference <sup>a</sup>	Easting difference <sup>b</sup>	Actual error <sup>c</sup>	Squared error <sup>d</sup>
1	10	12	15.62	244.00
2	8	15	17.00	289.00
3	6	13	14.32	205.00
4	7	9	11.40	130.00
Sum of squared error <sup>e</sup>				868.00
Mean square error <sup>f</sup>				217.00
Root mean square error (m) <sup>g</sup>				14.73

<sup>a</sup> Difference between test point and truth, in the Y-direction (North)

<sup>b</sup> Difference between test point and truth, in the X-direction (East)

<sup>c</sup> Straight line difference between reference point and known position  $(\text{Northing difference}^2 + \text{Easting difference}^2)^{0.5}$

<sup>d</sup> Actual error squared

<sup>e</sup> Sum of the squared errors

<sup>f</sup> Average of the squared errors

<sup>g</sup> Square root of the average of the squared errors

### Whitehall Forest Case Study #1

In Danskin et al. (2009), a number of mapping-grade and recreation-grade GPS receivers were tested in southern hardwood forest conditions. Some of the GPS units were WAAS-enabled, and this technology was compared to raw data collected from the receivers and data that was differentially corrected after data collection had concluded.

The results of this study indicate that the effect of slope position, vegetative cover, and interactions between these factors cause a high amount of variability in the performance of recreation-grade receivers tested.

Results include the following:

- The mapping-grade receiver provided the highest level of accuracy achieved.
- Differential correction of this data showed greater improvements than real-time differential correction with WAAS.
- WAAS, when enabled, produced more favorable results for those GPS receivers that had the capacity to utilize this new technology.
- Results demonstrated that some of the recreation-grade receivers performed well.

## Whitehall Forest Case Study #2

In Bettinger and Fei (2010), a single recreation-grade was tested in southern hardwood forest conditions and southern pine forest conditions over the course of an entire year. WAAS was not enabled.

The results of this study indicate that the most important and statistically significant effect was due to differences in forest type (horizontal accuracy in old pine < hardwood < young pine).

Results include the following:

- The horizontal positions recorded by the recreation-grade receiver were 7-12 meters off, on average. Worst-case, they were over 40 m off.
- Time of year did not influence the accuracy of the data collected by the receiver.
- Air temperature, relative humidity, and solar wind speed did not have an effect on the accuracy of the data collected by the receiver.

### 4.11 GPS TEST SITE AT THE WHITEHALL FOREST

A GPS test site was installed on the University of Georgia Whitehall Forest. Twenty-seven points, referenced by brass survey caps mounted on rebar and surrounded by approximately 6 inches of cement, were placed in the ground. Twenty-three of these points are in a predominately hardwood forest on north and east slopes. Three other points (for a total of 30 points) are located in open fields and were the subject of four hours of data collection with survey-grade GPS receivers. These three points were used to tie the twenty-seven forested points to a control network. The final coordinates of the points have a horizontal positional precision of approximately 2 cm.



A number of equipment tests have been performed using the test site. More information on the GPS test site can be obtained from [http://gps-test-site.uga.edu/UGA\\_GPS.html](http://gps-test-site.uga.edu/UGA_GPS.html).

## 4.12 TYPES OF GPS RECEIVERS

GPS receivers can be categorized by measurement accuracy and price into three broad grades or categories: survey, mapping, and recreation-grade.



The most accurate and most expensive GPS receivers are the survey-grade receivers. When used correctly, these receivers can calculate positions to within one cm of true location. They have a relatively high cost (greater than \$10,000), however, and use in natural resource applications has been very limited due to the delicate nature of the equipment and the requirement for sustained satellite reception.

Mapping-grade GPS can be purchased for between \$1,500-10,000 depending on features and manufacturer. Some mapping-grade receivers contain software for real-time differential correction. The manufacturers' estimates of positional accuracy are 1-5 m depending on the receiver configuration and mapping application.

Recreation-grade GPS receivers are the least accurate, yet most affordable of the GPS receivers, and cost between \$50-750. Some of these receivers do not allow operators to set minimum thresholds for satellite signal quality through the masks described earlier. While most recreation-grade receivers allow storing measurements individually, there is usually a limit (e.g., 500 points). Accuracy in forested settings has been reported to be between 5 and 15 m

All recreation-grade GPS receivers have the antenna embedded within the GPS unit. Most mapping-grade and survey-grade GPS receivers also have an antenna embedded within the unit, however, many allow the use of external antennas which are either of higher quality, or can be extended above localized obstructions.



### 4.13 FINAL THOUGHTS

Most people consider measurements collected from GPS to be accurate to the "sub-meter." However, when using GPS in the field, under trees, they generally are not. Visual inspection of the GPS data is crucial to ensure that the quality of measurements, when transferred to GIS, remains high.

Recreation-grade GPS receivers are generally not used to create GIS data that will be integrated with an organization's "official" GIS databases. The quality of data is generally not high enough to allow this. Recreation-grade GPS receivers are valuable for some navigational purposes, but not for locating precise positions in the field.

As a result, mapping-grade GPS receivers are used to develop data for the creation and update of GIS databases. Given the ability of GPS users to change the PDOP, SNR, and elevation masks within mapping-grade GPS receivers, many organizations have created a list of standards or protocols for field data collection activities. Adherence to the standards or protocols is important, since data collected outside the range of suggested parameters may be simply discarded.

When considering the use of GPS to make a map, develop a database, or simply to navigate, a number of questions should be considered prior to using the technology.

1. When is the best time to collect the data? One mistake that is sometimes made by novice GPS users is to assume that a GPS signal will be available with the appropriate PDOP values at all times of the day, which may not be the case. Mission planning will help you understand the best time of day to collect data.
2. How much time will I need to collect the data? Time may be needed to drive to the field, to assemble the GPS equipment, to set the parameters (PDOP, SNR, file name), and to navigate to

the place where data collection will begin. At the end of the day, time will be required to disassemble the equipment, travel back to the office, and perhaps download data to a computer.

3. Do I need to differentially correct the data? As noted earlier, differential correction is probably needed in almost every case in natural resource management where accuracy of positions is a concern.

4. Will the base station have data covering my time period? You need to check the availability of base station data because it may not have been collected continuously.

5. Do I need to remove multipath error? Manual deletion of some points or vertices may be needed even after differential correction.

6. Will the GPS data be integrated with other GIS data? Do the features need to connect? These concerns relate to the maintenance and management of GIS data. For example, when mapping streams or roads with GPS, the data may be integrated with an existing streams or roads GIS database. The physical connection of the new data to the existing data may be important from both an analytical point of view as well as a mapping and display point of view.

#### 4.14 REFERENCES

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## 5.0 GEOGRAPHIC INFORMATION SYSTEMS

It is commonplace to see geographic information systems (GIS) used by natural resource managers to help make decisions. In fact, many entry-level natural resource jobs require some knowledge and experience with GIS.

A GIS consists of the tools that allow one to *collect, organize, manipulate, analyze, and display* geographic information.

A GIS can be defined by how it is used (e.g., a land information system or a natural resource management information system), by what it contains (points, lines, polygons, or raster grid cells), by its capabilities, or by its role in an organization (i.e., a map production system, a spatial analysis system).

The core component of GIS are the spatial databases that contain geographically-referenced features and associated attributes.

The two main types of data are vector (points, lines, polygons) and raster (grid cells). Vector data are located and stored using *X, Y* (easting, northing) coordinates that define specific points or changes in direction or lines or edges of polygons. This type of data is best used to represent roads, streams, timber stands, inventory points, and other similar things of interest. Raster data are perhaps best described as a sheet of grid cells, located and stored using row and column information. Aerial imagery, satellite imagery, and continuous environmental data such as precipitation and temperature levels are usually stored as raster databases.

## 5.1 A VERY BRIEF HISTORY OF GIS

Spatial data have been collected and maintained for millennia, with records of property boundary surveys for taxation purposes in Egypt dating back to about 1400 BC. It is only within the past 40 years, however, that society has learned how to digitally capture, maintain, and analyze spatial data (Wing and Bettinger 2008).

The term “geographic information system” was first used in the 1960’s. During this time, several organizations in the United States, including the U.S. Geological Survey and the U.S. Department of Agriculture’s Natural Resource Conservation Service, began to create GIS databases.

To manage and analyze spatial data, Canadian and U.S. organizations began to develop software programs in the 1960’s, one of the first was the Canada Geographic Information System (CGIS). The Environmental Systems Research Institute (ESRI) was founded in 1969, where ArcGIS, the most widely used GIS software program in natural resource management, is developed.

GIS became a common tool for natural resource management in the late 1980’s and early 1990’s as personal computers became more affordable and GIS software became more user-friendly.

We are fortunate today to have a variety of GIS software programs from which to choose. These include:

ArcGIS (ArcMap)	Erdas Imagine
qGIS	Idrisi
GeoMedia	MapInfo Pro

Upon graduation, you should be proficient in using this type of software, and have a high level of competency for making a good map (like the one on [page 109](#)).

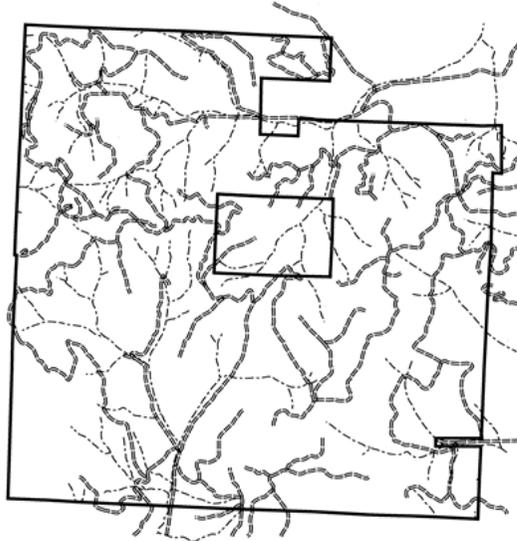
Whether you learn ArcGIS, qGIS, GeoMedia, or MapInfo Pro, you should not be concerned that the organization at which you have just been hired employs one you have not used. The basic skills developed by learning one of these can be transferred to the others. While the software programs are admittedly different, once you learn how to use one type of GIS software, you should have enough confidence to be able to quickly learn another.

Appendix G provides some instruction on making a map using ArcMAP.

## **5.2 REFERENCES**

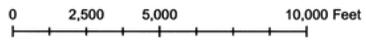
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# Brown Tract Roads and Streams



### Legend

- Roads
- ... streams



Map Created By: Philip Brown March 8, 2007

## 6.0 TREE AND WOOD-RELATED MEASUREMENTS

In order to make informed decisions regarding the management of natural resources, you need adequate resource-related information.

Much of the information we use to make decisions requires direct measurement of various resources. This book relates to a course on land and tree measurements. We have previously discussed land measurements, and now we turn our attention to the measurement of trees.

Several common measurements are used to characterize the structural condition of trees. These include measuring their girth, their length, and their age.

Characterization of the condition of a stand of trees will enable you to determine its value for wildlife habitat and economic objectives.

After completing this Chapter, you will be able to:

1. Understand how one would appropriately measure a tree's diameter, height, and age.
2. Calculate, from tree measurements, the basal area of a tree and estimate the basal area of a stand of trees.
3. Understand how one would determine the volume and weight of trees and down wood based on field measurements.

## 6.1 DIAMETER AT BREAST HEIGHT

The most common measure of the girth of live trees is the diameter at breast height. Breast height is considered to be a place on a tree that is 4.5 feet (1.37 meters) above ground level. We commonly use the acronym "DBH" to represent diameter at breast height measurements.

Specialized devices are used to provide these measurements directly based on the circumference of a tree bole.

Since all trees are not uniform, there are standards which one should use to appropriately measure the DBH of trees. These include the following:

1. Always measure DBH from the uphill side of a tree.
2. Do not measure over limbs, vines, or other objects that are not part of the main tree bole.
3. If a tree leans, measure the DBH perpendicular to the lean.
4. If a tree forks below DBH, each fork is considered a separate tree.
5. If a tree forks below, but near DBH, each fork is considered a separate tree, and the DBH measurement should be made a foot or so above the fork.
6. If a tree forks above DBH, it is considered one tree.
7. If a tree has an unusual bulge around DBH, make the DBH measurement a foot or so above the bulge.
8. If a tree has a bottleneck, such as a cypress tree, make the DBH measurement a foot or so above the bottleneck.

The devices that one could use to measure a tree's DBH include:

- DBH tape. To measure DBH, you physically measure the circumference of a tree bole. What you are actually interested in obtaining, however, is the diameter. DBH tapes convert the circumference measurement to diameter using the formula  $D = C/\pi$  and report DBH in "diameter equivalents of

circumference.”

To use a DBH tape, wrap the tape around a tree at the appropriate height on the tree's bole. One end of the tape will indicate "0." Where the "0" measurement overlaps values on the other end of the tape is the DBH measurement. Most DBH tapes that we used in the southern U.S. are calibrated to provide DBH in inches and tenths of inches. Other DBH tapes are metric.

A bark hook may be provided on a DBH tape to facilitate measuring larger diameter trees.



Tapes have the tendency to slip down the bole on the back side of the measurement, so care must be placed in making sure that the tape is snug against the bole, at 4.5' above ground, and does not contain branches or slack.

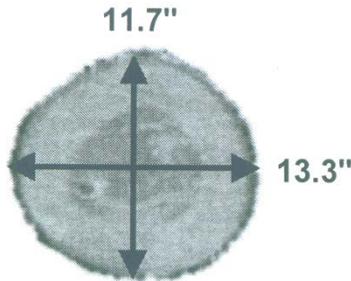
If a steel diameter tape is level and pulled tight, it is the most consistent method of measuring DBH. However, when stems are elliptically shaped, measurements are likely to be positively biased; consequently, tape measurements are less accurate than caliper measurements when stems are not perfectly round.

- Calipers. Attach the caliper to the appropriate height on the tree's bole. Read the measurement from the scale provided on the arm of the caliper. Move 90° around the tree bole and

obtain another measurement. Average the two measurements to arrive at the DBH.

$$DBH = \left( \frac{DBH_1 + DBH_2}{2} \right)$$

Two measurements are needed because trees are not necessarily round.

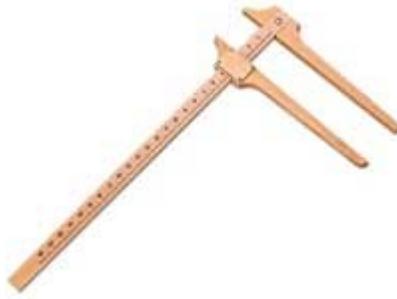


$$DBH = \left( \frac{11.7 + 13.3}{2} \right) = 12.5 \text{ inches}$$

If a tree bole is very elliptical in shape, calculate the geometric mean:

$$(DBH_1 \times DBH_2)^{0.5}$$

A sturdy, wooden, 3-foot long caliper is shown below.



- **Biltmore stick.** The Biltmore stick was developed by Dr. Carl Schenk, who was a director of the Biltmore Forest School. To use the Biltmore stick:
  1. Hold the Biltmore stick against the tree at the appropriate height on the tree's bole (4.5' above ground).
  2. Position the stick so that it is perpendicular to your sight.
  3. Position your eye such that it is 25 inches from the stick.
  4. Slide the "0" end of the stick so that it aligns with one edge of the tree's bole.
  5. Without moving your head, determine where the other side of the stick and the tree bole cross, and read the value presented on the stick. This is an estimate of the DBH.

If a tree bole is very elliptical in shape, take two measurements and calculate the DBH as noted above for calipers.

- Other devices, such as the Bitterlich sector fork, can be used to measure the DBH of trees. Here, the fork is positioned against the tree, and DBH is read directly using a sighting pin



and a scale. As with calipers, two diameters should be measured and averaged.

### Important points

1. The instruments described above can provide DBH estimates to the nearest 0.1 of an inch. When using these devices, you should record the measurements to the nearest 0.1 of an inch.
2. Some measurement systems require you to report DBH to the nearest 1 or 2 inch class. Therefore, you should, with experience, try to ocularly estimate the DBH class prior to actually measuring the DBH with one of the devices. At some point, you should get comfortable estimating DBH simply by looking at a tree, rather than physically measuring each tree with a DBH measuring device.

## 6.2 BARK THICKNESS

Tree stems are comprised of both wood and bark, and normal DBH estimates include the thickness of the bark. At some point, particularly from a commodity production point of view, you may find that an estimate of only the woody portion of a tree bole is needed.

- Normal DBH estimates are "outside bark" or "d.o.b."
- DBH estimates where bark is excluded are "inside bark" or "d.i.b."

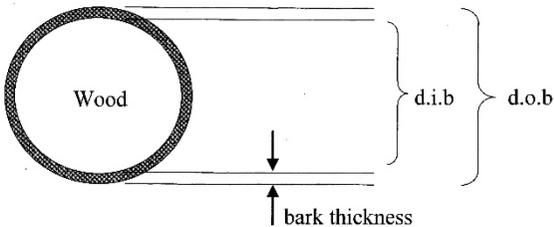
A bark gauge can be used to estimate the thickness of the bark.



To use a bark gauge, punch it into the tree at DBH. The sharp end of the gauge will pass through the bark and stop when it hits the wood (cambium). Measure the thickness of the bark by reading the scale on the stem of the gauge.

To estimate inside bark DBH, multiply the thickness of the bark by 2, and subtract the value from the outside bark DBH.

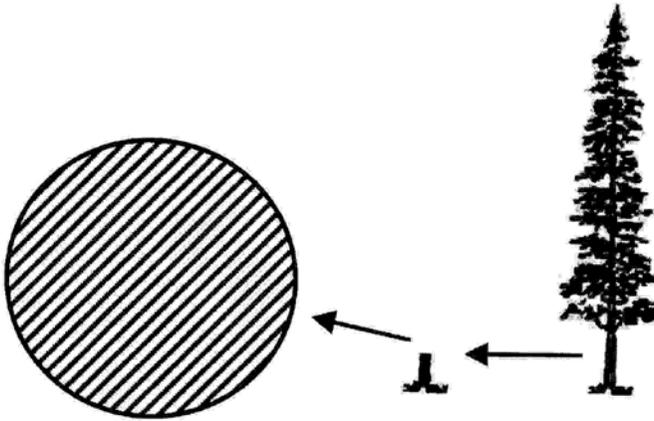
$$\text{d.i.b.} = \text{DBH} - (2 \times \text{bark thickness})$$



### 6.3 BASAL AREA

Basal area is the cross-sectional surface area of a tree at DBH. It is measured perpendicular to the stem of a tree. This measure of tree density is used in the forestry, wildlife, and fisheries professions. We commonly use the acronym "BA" to represent basal area estimates.

To envision basal area, imagine that a tree has been cut at DBH. If you were to measure the surface area of the top of the stump, and convert it to square feet (or in other parts of the world, square meters), you would arrive at the commonly-communicated measure of basal area.



To calculate basal area, assume that a tree bole is perfectly round. The area of a circle is:

$$\text{Area} = \pi r^2$$

DBH is the diameter of a tree, so the radius ( $r$ ) of a tree is one-half of the DBH, or  $(\text{DBH} / 2)$ .

Substituting  $(\text{DBH} / 2)$  into the area equation, the equation then

becomes:

$$\text{Area} = \pi \left( \frac{\text{DBH}}{2} \right)^2 = \pi \left( \frac{\text{DBH}^2}{4} \right)$$

Since DBH is measured in inches in the southern U.S., and since we want BA in square feet, we divide everything by 144 (the number of square inches in a square foot).

$$\text{Area} = \left( \frac{\pi \left( \frac{\text{DBH}^2}{4} \right)}{144} \right)$$

The condensed version of this equation, and one that should be committed to memory is:

$$\text{BA (square feet)} = 0.005454 \text{ DBH}^2$$

**Example** What is the basal area of a tree with a DBH of 13.6 inches?

$$\text{BA} = 0.005454 (13.6)^2 = 1.01 \text{ square feet}$$

### Important points

1. When the basal area of all trees on an acre are added together, we obtain the basal area in *square feet per acre*.
2. We need to estimate the BA in *square feet per acre* because this measure is what is commonly communicated amongst natural resource professionals, and what is commonly used in some habitat suitability models.

## 6.4 HEIGHT AND CROWN-RELATED MEASUREMENTS

As resource managers, we are interested in various measurements of the height of both live and dead trees (snags). Some of the vertical height measurements include:

- Total tree height: length of tree from ground level to the tip of the tree.
- Merchantable tree height: length of tree from ground level to some pre-determined upper-stem diameter of the bole.
- Crown length: length of crown from the base to the tip.
- Crown ratio: crown length / total tree height.

In addition, two other horizontal crown-related measurements may be of interest to natural resource managers:

- Crown width: distance from one edge of the crown to another.
- Crown area: the area on the ground that the crown covers.

A number of tools can be used to measure crown-related characteristics, including:

1. Height poles. These are extendable plastic poles that allow one to measure the height of a tree up to about 40 feet. They require two people to operate effectively, one to hold the pole, and another to indicate when the pole has reached the appropriate place on the tree bole to measure the height.

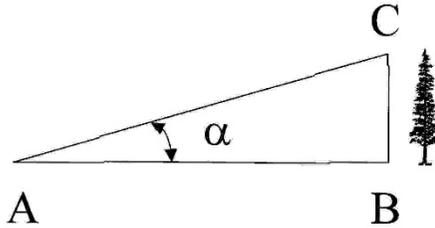
2. Hypsometers. These are devices that you hold in your hand to estimate tree height. Only one person is needed to use them effectively. These tools use trigonometric relationships to measure tree height. Tree height is estimated based on a known distance and angle(s) between the user and one or more points on the tree. Suunto clinometers, Haga altimeters, and Abney levels require the user to stand a known distance from the tree, measured by pacing or taping. These devices measure the

angle(s) between the user and the tree and estimate height based on trigonometric relationships. Failure to accurately measure the horizontal distance between the user and the tree can cause major errors in height estimates made with clinometers and similar devices. Newer hypsometers may use lasers or ultrasound waves to measure the distance between the user and the tree. These devices then measure the angle(s) between the user and the tree and estimate height using trigonometric relationships.

Regardless of what type of hypsometer is used, the user should stand approximately one tree height away from the tree to facilitate accurate measurement of angle(s).

## 6.5 TREE HEIGHT MEASUREMENTS USING TRIGONOMETRY

Using the following conceptual model, assume that we need to determine the length of the BC side of the right triangle.



From trigonometry we know that the tangent of the angle  $\alpha$  is equal to the opposite side over the adjacent side of the triangle.

BC is the opposite side  
 AB is the adjacent side  
 AC is the hypotenuse

Therefore the tangent of angle  $\alpha$  is:

$$\tan \alpha = \left( \frac{BC}{AB} \right)$$

Since BC is what we need, and since we can measure AB (by pacing away from the tree) and the angle  $\alpha$  (using a hypsometer), we can rearrange the equation to solve for side BC (the tree height).

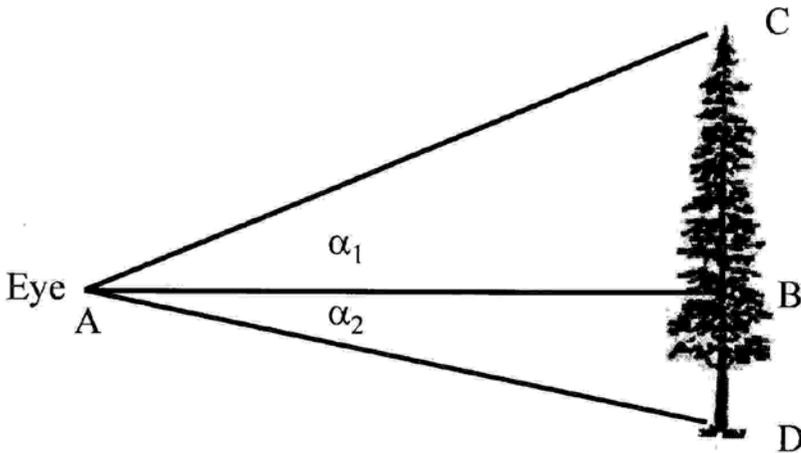
$$BC = (AB) (\tan \alpha)$$

**Example** What is the height of a tree when you are standing 100 feet away, and you determine that the angle to the tip is  $36^\circ$ ?

$$\text{Height} = (100 \text{ feet}) (\tan 36^\circ) = 73 \text{ feet}$$

This simple model, of course, only works when the angle from your line of sight to the base of the tree is flat ( $0^\circ$ ), which rarely occurs in nature. Therefore, there are three general cases related to tree height measurements, and each assumes that two right triangles are present.

CASE A. Eye is above the base of the tree, and below the tip of the tree



Here, there are two right triangles: ABC and ABD. Triangle ABC is associated with angle  $\alpha_1$ , and triangle ABD is associated with angle  $\alpha_2$ .

The height of the tree can be estimated by calculating the values for sides BC (from the first triangle) and BD from the second triangle:

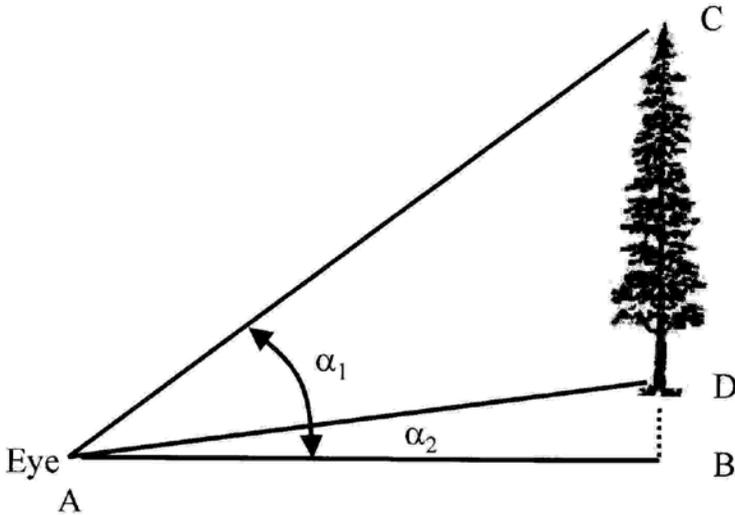
$$\text{Height} = (AB) (\tan \alpha_1) + (AB) (\tan |\alpha_2|)$$

**Example** You walk away from a tree 66 feet, and find that the angle to the tip is  $26^\circ$ , and the angle to the base is  $-2^\circ$ . What is the tree height?

$$\text{Height} = (66 \text{ feet}) (\tan 26^\circ) + (66 \text{ feet}) (\tan |-2^\circ|) = 34.5 \text{ feet}$$

CASE B. Eye is below the base of the tree, and below the tip of the tree

In this case, you find yourself standing on the slope of a hill well below the base of the tree.



Here, the two right triangles are: ABC and ABD. The opposite side of triangle ABD represents the vertical height to the base of the tree from a horizontal line that extends from your eye to some place underground, beneath the tree. Triangle ABC is associated with angle  $\alpha_1$ , and triangle ABD is associated with angle  $\alpha_2$ .

The height of the tree can be estimated by calculating the values for sides BC (from the first triangle) and BD from the second triangle:

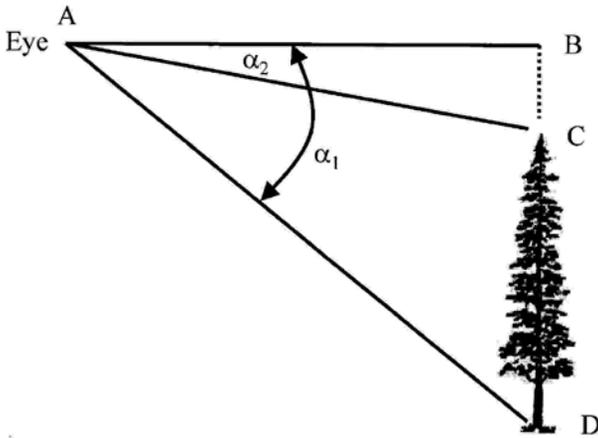
$$\text{Height} = (AB) (\tan \alpha_1) - (AB) (\tan \alpha_2)$$

**Example** You walk away from a tree 66 feet, and find that the angle to the tip is  $34^\circ$ , and the angle to the base is  $5^\circ$ . What is the tree height?

$$\text{Height} = (66 \text{ feet}) (\tan 34^\circ) - (66 \text{ feet}) (\tan 5^\circ) = 38.7 \text{ feet}$$

CASE C. Eye is above the base of the tree, and above the tip of the tree

In this case, you find yourself standing on the slope of a hill well above the tip of the tree.



Here, the two right triangles are: ABC and ABD. The opposite side of triangle ABC now represents the vertical height to the tip of the tree from a horizontal line that extends from your eye to some place well above the tip of the tree. Triangle ABD is associated with angle  $\alpha_1$ , and triangle ABC is associated with angle  $\alpha_2$ .

The height of the tree can be estimated by calculating the values for sides BC (from the first triangle) and BD from the second triangle:

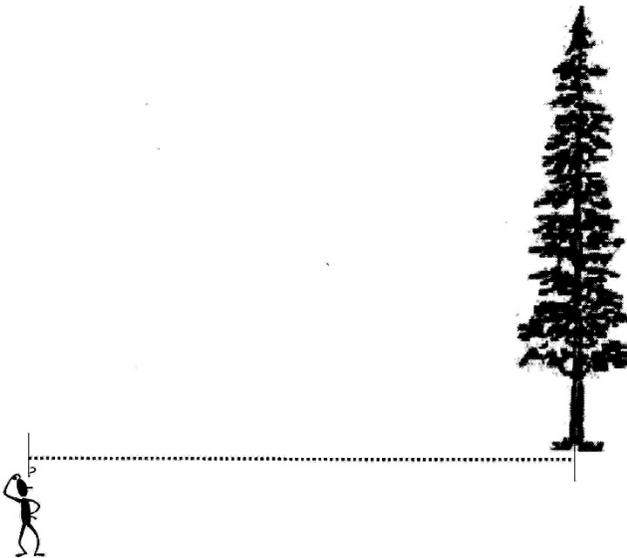
$$\text{Height} = (AB) (\tan |\alpha_1|) - (AB) (\tan |\alpha_2|)$$

**Example** You walk away from a tree 66 feet, and find that the angle to the tip is  $-10^\circ$ , and the angle to the base is  $-41^\circ$ . What is the tree height?

$$\begin{aligned} \text{Height} &= (66 \text{ feet}) (\tan |-41^\circ|) - (66 \text{ feet}) (\tan |-10^\circ|) \\ &= 45.7 \text{ feet} \end{aligned}$$

### Important points

1. These three cases can also be extended to the estimation of merchantable tree height or crown length. The appropriate place on the tree's bole must be used to develop the measurements.
2. A relatively accurate measurement of the distance from the tree to the place where you make the angle measurements is necessary. This distance is made from the center of the tree out to where you will be standing (not the front side nor the back side of the tree).



3. If a tree leans, make the measurements perpendicular to the lean.
4. One common error in these calculations is when you used radians rather than degrees in your calculators. You need to determine how to switch your calculator mode from the use of radians to the use of degrees when performing the computations.

## 6.6 TREE HEIGHT MEASUREMENTS USING A SUUNTO CLINOMETER

A clinometer can not only allow you to measure the slope of land, but also the height of objects such as trees. No trigonometry is needed to estimate heights, since the device is calibrated to take the trigonometric relationships into account.



To use the clinometer, hold it up to your eye as oriented in the picture above, and look through the black eye piece. Keep both eyes open. One eye will be viewing the scales inside the clinometer, the other will be viewing the object of interest.

When looking into the Suunto clinometer, you will see that it has two scales:

1. The *percent scale* (the left-hand side scale). This is the scale that you would normally use to measure ground slope. For estimating the height of objects, it is calibrated under the assumption that a person stands 100 feet away from the object. Therefore, the calibration distance is 100 feet. Under normal circumstances (i.e., you are standing 100 feet from a tree), you simply read the internal scale when viewing the upper part of the stem of a tree, then read the scale when viewing the lower part of the stem of a tree. With these two measurements, you can estimate the height of a tree. If you

use this scale, but do not stand 100 feet from the center of the tree, the following equation can be used to convert each of the two measurements to the values needed to estimate tree height:

$$Value = \left( \frac{\text{Distance from tree}}{\text{Calibration distance}} \right) \text{clinometer reading}$$

2. The *topographic scale* (the right-hand side scale). This scale is calibrated under the assumption that a person stands 66 feet (1 chain) from an object when measuring its height. Therefore, the calibration distance is 66 feet. Under normal circumstances (i.e., you are standing 66 feet from a tree), you simply read the internal scale when viewing the upper part of the stem of a tree, then read the scale when viewing the lower part of the stem of a tree. With these two measurements, you can estimate the height of a tree. If you use this scale, but do not stand 66 feet from the center of the tree, the following equation can be used to convert each of the two measurements to the values needed to estimate tree height:

$$Value = \left( \frac{\text{Distance from tree}}{\text{Calibration distance}} \right) \text{clinometer reading}$$

In either case, you must obtain two measurement values:

- The top (or merchantable height) of a tree
- The base of a tree

To estimate tree height, two rules of thumb should be used:

1. If one measurement is positive and one measurement is negative, add the positive measurement to the absolute value of the negative measurement to arrive at an estimate of the tree height.

2. If both measurements have the same sign (positive or negative), subtract one measurement from the other to arrive at an estimate of the tree height. The absolute value of the measurements must be used when both are negative.

As this relates to the "cases" described in Section 6.5:

CASE A: One measurement will be positive, one measurement will be negative.

Tree height = Positive measurement (the top of the tree)  
+ |Negative measurement (the base of the tree)|

CASE B: Both measurements will be positive.

Tree height = Positive measurement (the top of the tree)  
- Positive measurement (the base of the tree)

CASE B: Both measurements will be negative.

Tree height = |Negative measurement (the base of the tree)|  
- |Negative measurement (the top of the tree)|

**Example** Using the topographic scale of the Suunto clinometer, you walk away from a tree 66 feet, and record two measurements: +55 (top of the tree), -7 (base of the tree). How tall is the tree?

This is Case A, and since the topographic scale was calibrated to a 66 foot viewing distance, the tree is  $55 + |-7|$ , or 62 feet tall.

**Example** Using the topographic scale of the Suunto clinometer, you walk away from a tree 33 feet, and record two measurements: +50 (top of the tree), -2 (base of the tree). How tall is the tree?

This is Case A, and since the topographic scale was calibrated to a 66 foot viewing distance, the tree is

$$\text{Tree height} = \left( \frac{33 \text{ feet}}{66 \text{ feet}} \right) (50 + |-2|) = 26 \text{ feet}$$

**Example** Using the percent scale of the Suunto clinometer, you walk away from a tree 100 feet, and record two measurements: +85 (top of the tree), +7 (base of the tree). How tall is the tree?

This is Case B, and since the topographic scale was calibrated to a 100 foot viewing distance, the tree is 85 - 7, or 78 feet tall.

**Example** Using the percent scale of the Suunto clinometer, you walk away from a tree 150 feet, and record two measurements: -2 (top of the tree), -80 (base of the tree). How tall is the tree?

This is Case C, and since the topographic scale was calibrated to a 100 foot viewing distance, the tree is

$$\text{Tree height} = \left( \frac{150 \text{ feet}}{100 \text{ feet}} \right) (|-80| - |-2|) = 117 \text{ feet}$$

### Important points

1. One or more of the clinometer measurements could be negative (below the horizontal line from your eye to the tree). Pay attention to the sign (+ or -) on the scale.
2. Try not to move your head when you switch from viewing the base to the top of the tree (or vice versa).
3. You must stand at least as far away from a tree as the tree is tall. For a 100 foot tree, you should stand at least 100 feet away. For a 30 foot tree, you don't need to stand 100 feet away.

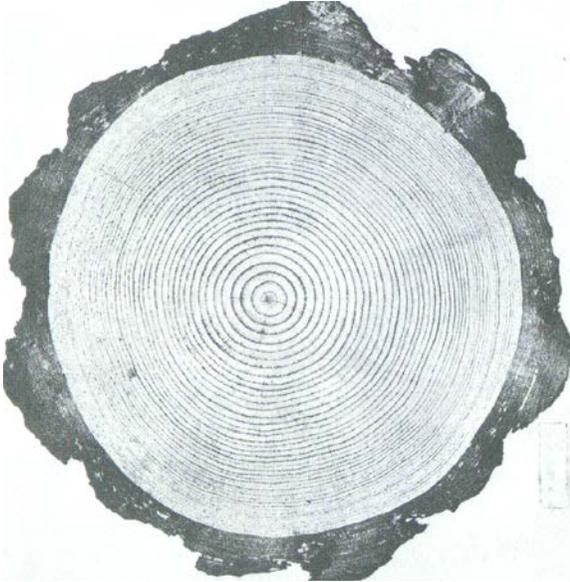
## 6.7 TREE AGE

Tree or stand age is a useful measurement for describing forests, and is often helpful in predicting future growth and yield of trees.

Age can be defined in several ways:

- Elapsed time since germination of a seed
- Elapsed time since budding or sprouting
- Elapsed time since planting or seeding
- Elapsed time since the tree was 4.5 feet tall

In temperate climates, most trees record their history of growth in the form of annual growth rings.



(Forest Products Laboratory 1987)

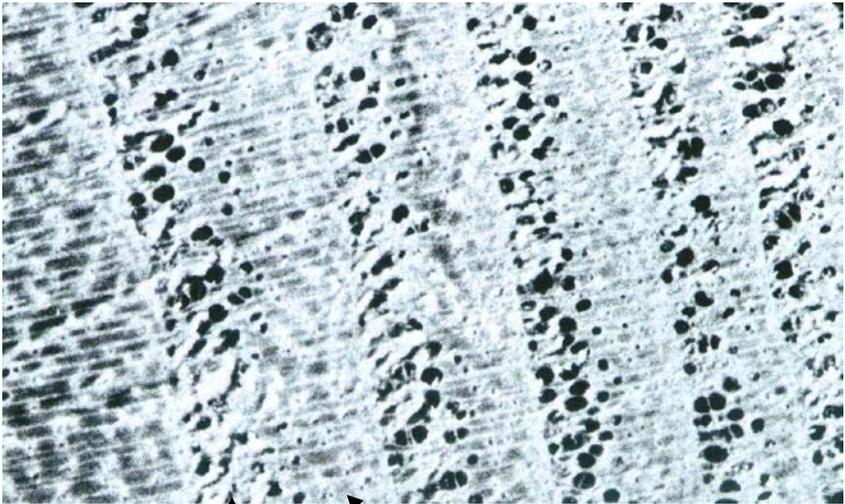
Each annual ring is usually distinctive, and made up of earlywood (the light colored, fast growing wood developed in the spring and summer), and latewood (the dark colored, slow growing wood developed in the fall).

## Pine



earlywood      latewood

## Hickory

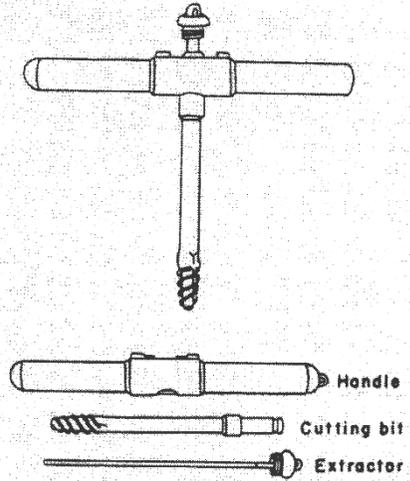


earlywood      latewood

The tools one can use to estimate tree age include increment borers, management history (as recorded by some management organizations), and branch whorls (for some trees).

### Increment borer

An increment borer is composed of three pieces: the handle, the cutting bit, and the extractor (or spoon).



Increment borers are screwed into a tree at DBH, to the point where the borer itself is past the mid-point of the tree bole (i.e., through the pith). When extracted, the core is about pencil-sized.



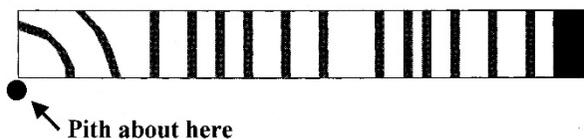
If the core indicates that the pith was "hit," you can count the rings from the pith to the cambium or bark to arrive at an estimate of the age of a tree once it was taller than DBH. You will need to add a few years to this estimate to arrive at total tree age.

Using an increment borer:

1. Place the cutting bit into the handle.
2. Locate DBH on a tree.
3. Find a place in the crack of the bark to begin the coring process. Starting on the outside of a large piece of bark will be more difficult.
4. Very slowly and carefully, and with some pressure, screw the increment borer into the tree.
5. Once the increment borer is entering into the tree, you have one last attempt to adjust it so that it is aimed at the pith.
6. Screw the increment borer into the tree, to a point past where you think the pith is located.
7. Slide the extractor into the cutting bit.
8. Rotate the borer outward one full rotation.
9. Pull out the core.

Important points

1. If the core was not "hit," look for rings near where the core should be - they should be more rounded than the normal tree rings in a core. In this case, rather than re-coring the tree, you might estimate where the pith is located and further estimate how many rings may be absent from the core.



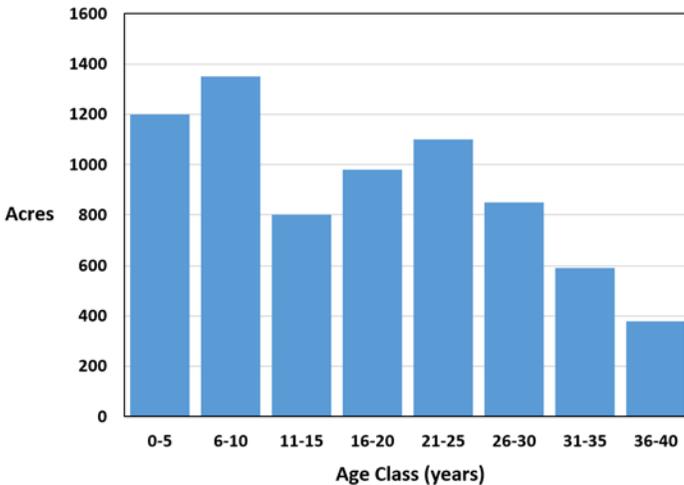
2. The extractor (spoon) is the most commonly damaged piece of the increment borer. They are damaged when sliding the extractor into the cutting bit. Slide the extractor in only as far as reasonably possible - do not force it or it may bend and break. Extractors cost \$15-25.
3. If parts of the core are stuck in the bit, remove them carefully. Try not to damage the sharp end of the bit, nor to cut yourself.

## 6.8 AGE CLASSES AND AGE CLASSIFICATIONS

### Age classes

There are a number of occasions in natural resource management in which we communicate "age classes" to each other. These may be the age of a stand (not an individual tree) grouped into 1-, 5-, or 10-year classes.

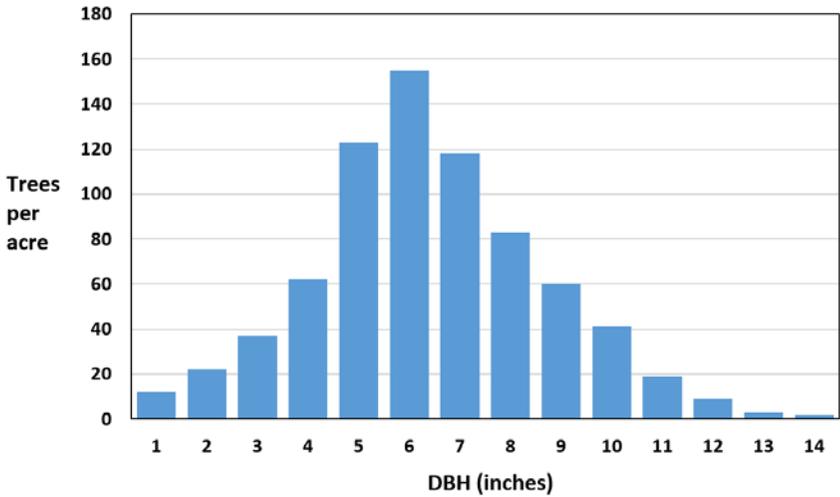
Age classes are used in developing age class distributions - acres by age class.



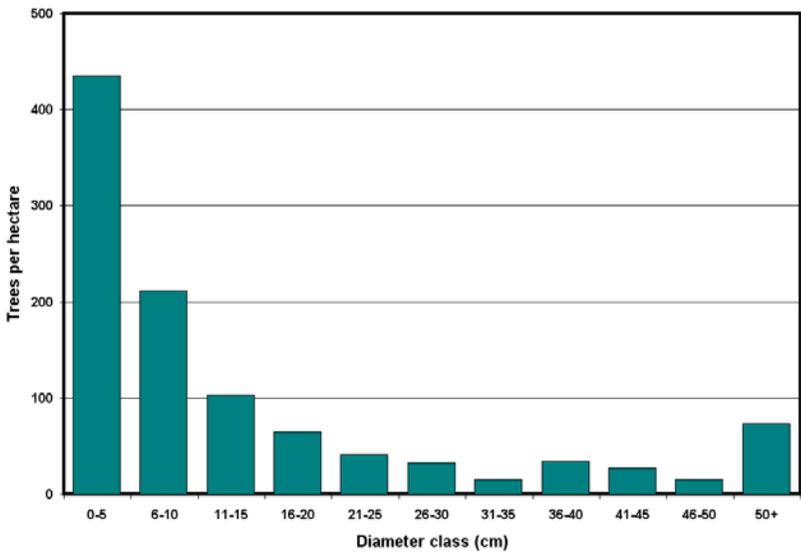
### Age classifications

Two basic age classifications are used to describe stands of trees: even-aged stands, and uneven-aged stands.

Even-aged stands are ones where the range of tree ages are generally within about 20% of the average stand age. When developing a diameter distribution (trees per acre by DBH class), the distribution should be approximately normal. The graph at the top of the next page represents the diameter distribution for an even-aged stand of trees.



Uneven-aged stands are ones where there are three or more distinct age classes. When developing a diameter distribution (trees per acre by DBH class), the distribution should be approximately reverse J-shaped.



## 6.9 CROWN CLASSES

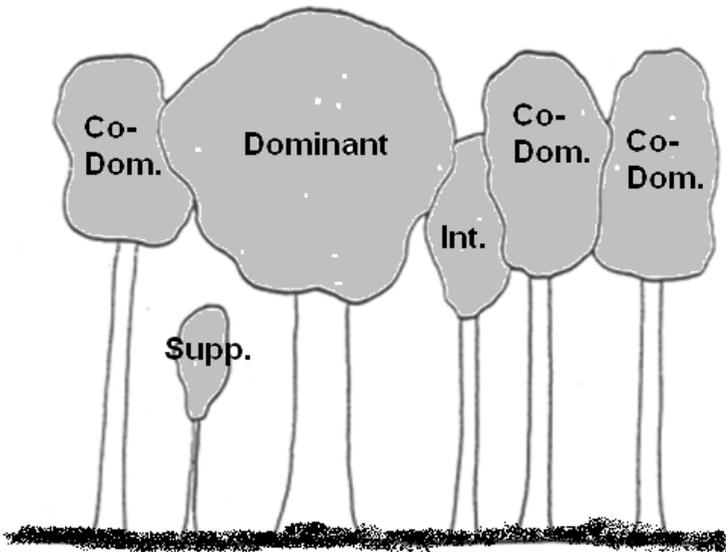
Four general classes of crowns are used to classify the position of a tree in a stand. These are related to the age of the tree, and the amount of growing space each tree has been provided.

*Dominant trees* - Trees with crowns that receive full sunlight from above and on the sides. These trees extend above the general crown canopy.

*Co-Dominant trees* - Trees with crowns that receive full sunlight from above. These trees extend into the upper crown canopy.

*Intermediate trees* - Trees generally shorter than dominant or co-dominant trees, but extending up into the upper crown canopy. They receive little or no sunlight from above.

*Suppressed trees* - Trees entirely below the upper crown canopy.



## 6.10 STANDING TREE VOLUME

The volume of a tree can include the wood in the main tree bole, the bark, the limbs, the roots, and the foliage. Generally, when we estimate the volume for southern U.S. trees, we are attempting to measure the volume of the main tree bole, and perhaps the bark associated with it.

Two general volume measures are used for trees: solid volume and board foot volume.

Solid volume includes cubic foot, cubic meter, and cord measurements.

1. A cubic foot is equal to a block of wood  $12'' \times 12'' \times 12''$ .
2. A cubic meter is equal to a block of wood  $1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$ . One cubic meter is equal to 35.32 cubic feet ( $3.281^3$ ).
3. A standard cord is  $4' \times 4' \times 8'$ , or 128 cubic feet of wood and air. The net volume (minus the air) is about 90 cubic feet. One cord is about 3.62 cubic meters ( $2.55 \text{ m}^3$  net).

A board foot is 12" long, 12" wide, and 1" thick. When estimating this volume, you take into account the kerf (sawdust) created when a board is milled (cut). A number of methods are used for estimating board foot volume - Scribner, International 1/4 rule, Doyle. These are investigated further in Forest Mensuration.

How can you determine the exact volume of an irregularly shaped solid such as a tree stem?

The most accurate way is to immerse the tree stem in water and determine how much water is displaced. The amount of water displaced is directly related to the volume of the tree stem. Unfortunately, this is difficult to do without first cutting the tree down.....

The next most accurate method may be to:

1. Measure the stem diameter in sections of 4 or 5 feet.
2. Assume some geometric shape for each stem section.
3. Calculate the volume of each section.
4. Sum the volumes to obtain the total stem volume.

However, this method is very labor-intensive and therefore not practical.

To obtain an approximate volume of a tree stem, natural resource managers use equations that are based on two field measurements: DBH and height.

Tree volume equations have been developed for most of the commercially important tree species in the U.S., as well as a variety of other common tree species. One of the classical volume models is the combined-variable equation:

$$volume = \alpha + \beta (DBH^2 H)$$

Here, both DBH and tree height (H) are used to estimate tree volume. More sophisticated models exist. The following models can be used to estimate volume for loblolly pine trees grown on cutover, site prepared land in the Coastal Plain of Virginia and North Carolina.

$$volume (ft^3, \text{ outside bark}) = 0.18658 + 0.00250 (DBH^2 H)$$

The associated board-foot volume equations that allow one to estimate the volume of the tree stem to a 6" top are (from Burkhart et al. 1987):

$$volume (board\ feet, \text{ inside bark, Scribner}) \\ = -29.7455 + 0.01888 (DBH^2 H)^{0.9521}$$

$$volume (board\ feet, \text{ inside bark, Doyle}) \\ = 3.2492 + 0.00003386 (DBH^2 H)^{1.5651}$$

$$\begin{aligned} \text{volume (board feet, inside bark, International 1/4)} \\ = -24.3816 + 0.005816 (\text{DBH}^2 \text{ H})^{1.0835} \end{aligned}$$

**Example** Assume that one of your relatives in Virginia has a loblolly pine tree on their property that is 24" DBH and 109 feet tall. How much cubic foot volume and board foot volume might this tree contain?

Using the Burkhart et al. (1987) equations, you could estimate the cubic foot, outside bark volume to be

$$\text{volume} = 0.18658 + 0.00250 (24^2 109) = 157.15 \text{ ft}^3$$

The associated board foot volume might be, depending on the board foot volume estimation rule that is used locally,

$$\begin{aligned} \text{volume (Scribner)} &= -29.7455 + 0.01888 (24^2 109)^{0.9521} \\ &= 669 \text{ board feet} \end{aligned}$$

$$\begin{aligned} \text{volume (Doyle)} &= 3.2492 + 0.00003386 (24^2 109)^{1.5651} \\ &= 1,097 \text{ board feet} \end{aligned}$$

$$\begin{aligned} \text{volume (International 1/4)} &= -24.3816 + 0.005816 (24^2 109)^{1.0835} \\ &= 894 \text{ board feet} \end{aligned}$$

### Important points

1. Tree volume equations for softwood and hardwood trees common to Georgia and the southeastern U.S. are noted in Appendix D.
2. The order in which the computations are made in these equations may affect the outcome. Follow the order of precedence that is suggested in making the computations.

## 6.11 STANDING TREE WEIGHT

In natural resource management we commonly measure the size of trees by both their volume and their weight. Many forestry companies rely on weight estimation in the decision to buy or sell timber.

The exact weight of a tree is an elusive measurement. If you wanted to know exactly how much a tree bole (without all of the limbs) weighed, you could place it on a mill scale, or other large scale, and obtain the value. However, this is not practical in operational activities. Therefore, we generally use weight equations to obtain estimates of stem weight. Weight equations generally require two field measurements: DBH and height.

*Green weight* is the weight of a freshly cut or fallen tree.

*Dry weight* is the weight of a tree, snag, or log after it has air dried. Dry weight is about 1/2 of the green weight.

As an example of a weight equation, the predicted green weight of total tree (wood, bark, foliage) for Coastal Plain southern pine could be estimated with the following equation (Clark and Saucier 1990) for trees  $\geq 5$  inches DBH:

$$\text{weight (pounds)} = 0.19821 (\text{DBH}^2)^{1.06419} (\text{H})^{0.91330}$$

**Example** Using the equation above, what is the weight of a 14.4" DBH, 70-foot tall southern pine tree?

$$\text{weight} = 0.19821 (14.4^2)^{1.06419} (70)^{0.91330} = 2,803 \text{ lbs}$$

### Important points

1. Standard tree weight equations for Georgia and the southeastern U.S. are noted in Appendix D.

## 6.12 TREE VOLUME AND WEIGHT EQUATIONS AND TABLES

There are three common types of volume and weight equations:

### 1. Standard volume or weight equations

Inputs are DBH and height.

Applicable to a particular species and region.

This type of equation is the one most often published.

### 2. Standard "form class" volume or weight equations

Inputs are DBH, height, and some measure of form or taper.

Applicable across a wide range of species and regions.

Usually used only for high quality products due to the time required to measure form.

### 3. Local volume or weight equations

Input is DBH alone.

Applicable to a particular species and stand (sometimes even a particular plot).

Typically constructed from a standard volume equation.

Volume or weight tables can be derived from these equations. An example of a standard weight table is shown on the next page. It was created from the following equation (from Clark and Saucier, 1990):

$$GW = 0.15179(DBH^2)^{1.02917} H^{0.96160} \exp(-2.77201 d^{4.78156} DBH^{-5.01343})$$

where GW = green weight of stem wood and bark to a  $d$  inch top outside bark. Here,  $d = 4$  inches.

DBH = diameter at breast height (in)

H = total tree height (ft)

DBH	Total height (ft)								
	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
5	39	57	75	93	111	129	146	164	181
6	83	123	162	201	239	277	315	353	391
7	132	194	256	317	378	439	499	559	618
8	184	271	358	443	528	613	697	780	864
9	241	355	469	581	692	803	913	1,022	1,131
10	303	448	591	732	872	1,012	1,150	1,288	1,425
11	372	549	724	898	1,069	1,240	1,410	1,579	1,748
12	447	660	870	1,078	1,285	1,490	1,694	1,898	2,100
13	528	780	1,029	1,275	1,519	1,762	2,003	2,244	2,483
14	616	910	1,200	1,488	1,773	2,056	2,337	2,618	2,897
15	711	1,050	1,385	1,716	2,045	2,372	2,697	3,021	3,343
16	813	1,200	1,583	1,962	2,338	2,711	3,083	3,452	3,820
17	921	1,361	1,794	2,224	2,650	3,073	3,494	3,913	4,330
18	1,037	1,531	2,019	2,502	2,981	3,458	3,932	4,403	4,873
19	1,159	1,712	2,257	2,797	3,333	3,866	4,396	4,923	5,448
20	1,288	1,903	2,509	3,109	3,705	4,297	4,886	5,472	6,055
21	1,425	2,104	2,774	3,438	4,097	4,752	5,403	6,051	6,696
22	1,568	2,315	3,053	3,784	4,509	5,230	5,946	6,659	7,369
23	1,718	2,538	3,346	4,147	4,942	5,731	6,517	7,298	8,076
24	1,876	2,770	3,653	4,527	5,394	6,256	7,114	7,967	8,816
25	2,040	3,013	3,973	4,924	5,868	6,805	7,738	8,665	9,589
26	2,212	3,266	4,307	5,338	6,361	7,378	8,388	9,394	10,396
27	2,390	3,530	4,655	5,770	6,875	7,974	9,066	10,153	11,236
28	2,576	3,805	5,017	6,218	7,410	8,594	9,771	10,943	12,110

This is a standard weight table because it requires inputs of DBH and height to use it. This is also an example of a merchantable weight equation because it provides estimates of weight for only a portion of the tree, not the entire stem.

Most often, we are interested in obtaining an estimate of the merchantable portion of the main stem. This is usually defined as that part of the stem up to a certain top diameter limit (e.g. 4" o.b., 6" o.b., etc.).

Two types of merchantable volume (weight) equations have been developed and are available for many species throughout the Southeast:

1. Separate equations (tables) for each top diameter limit.
2. Ratio equations that can be used for any top diameter limit.

Separate equations for each top diameter limit

Usually these equations will have a form similar to the following function:

$$V = B_0 + B_1 \text{DBH}^2 H$$

where  $V$  = volume or weight of the merchantable part of the stem

$B_0$  and  $B_1$  = coefficients unique to a given species and top diameter limit in a given region

DBH = diameter at breast height

H = height

This particular form of equation is called a combined variable equation because the DBH and H are combined into one variable which has only one coefficient multiplied by it (the  $B_1$ ).

**Example** Outside bark merchantable volume of old field loblolly pine (Tennessee, Alabama, and Georgia highlands):

$$V_2 = -0.1034 + 0.0026118 (\text{DBH}^2 H) \text{ (volume to a 2" top o.b.)}$$

$$V_3 = -0.1051 + 0.0026271 (\text{DBH}^2 H) \text{ (volume to a 3" top o.b.)}$$

$$V_4 = -0.6054 + 0.00266381 (\text{DBH}^2 H) \text{ (volume to a 4" top o.b.)}$$

**Example** Old field loblolly pine in the Georgia Piedmont:

$$GW_3 = -17.262 + 0.13819 (\text{DBH}^2 H)$$

$$GW_4 = -48.903 + 0.13905 (\text{DBH}^2 H)$$

### Ratio merchantable volume (or weight) equations

The general form of the equation is:

$$V_m = V_t (1 - b_1 (d^{b_2} / DBH^{b_3}))$$

where  $V_m$  = merchantable volume or weight to a top diameter of  $d$  inches

$V_t$  = total stem volume or weight

DBH = diameter at breast height

$b_1$ ,  $b_2$ , and  $b_3$  = coefficients specific to a particular species and region

In this formulation,  $b_1 (d^{b_2} / DBH^{b_3})$  represents the top part of the tree above the merchantable portion - the part which must be removed.

**Example** PMRC equations for site prepared slash pine plantations in Georgia and north Florida:

$$V_t = 0.0061645 DBH^{2.05779} H^{0.74679}$$

$$V_m = V_t (1 - 0.61529 (d^{3.66826} / D^{3.47361}))$$

Another common form of the equation is:

$$V_m = b_4 D^{b_5} H^{b_6} \exp(-b_1 (d^{b_2} / D^{b_3}))$$

In this variation, the total volume and the merchantable volumes are fit all at once. If  $d = 0$ , then all that is left is the part of the equation before "exp" which predicts the total volume. In this formulation,  $\exp(-b_1 (d^{b_2} / D^{b_3}))$  represents the ratio of the total volume that is merchantable.

**Example** Estimating the weight of wood in a tree between two merchantable heights. Say we have a 16 inch DBH, 80 foot tall southern pine and we are using the ratio merchantable weight equation presented at the beginning of Section 6.12. According to the table presented a few pages back, the merchantable weight to a 4" top is 3,083 pounds.

If we define the sawtimber portion of the tree as the portion below an 8" top (o.b.), the weight in that portion of the tree is:

$$GW = 0.15179 (16^2)^{1.02917} 80^{0.96160} \exp(-2.77201 8^{4.78156} 16^{-5.01343})$$

GW = 2,929 pounds.

The topwood, or pulpwood, in the tree is the difference between the weight to a 4" top and the weight to an 8" top. In this case the topwood is 3,083 - 2,929 = 154 lbs.

### Evaluating volume and weight equations

Which standard volume or weight equation should you use?

Ideally, one which was developed for your species, region, stand type, range of tree sizes, etc.

Even if you have a match, and especially if you don't, you should check out the equation you are using or the ones you are considering. This requires some effort. Here's how:

1. Find the actual volume (or weight) of a sample of trees across the range of tree sizes in your stands by felling them and obtaining volume (or weight) directly.
2. Calculate the predicted volume (or weight) using your equation(s). You now have an actual and at least one predicted value for each of your sample trees.

3. Take the actual volume (or weight) and subtract off the predicted volumes (or weights). The result is the error or residual - the amount that the prediction is off. Some of these should be positive and some negative; hopefully many of them are near zero.

4. Plot the residual values over DBH and also over H. You are looking for a uniform horizontal band around zero over the entire range of both DBH and H. If you don't see it, that means that your equation over-predicts in part of the range and under-predicts in other parts of the range of DBH and H. As a result, you should look for another equation.

### 6.13 TIMBER PRODUCTS IN THE U.S. SOUTH

Categorizing timber by product class allows effective management evaluation and planning. Particularly on private land, multiple-use goals from wildlife to aesthetics are usually dependent on timber management to fund projects. Total merchantable volume or weight is only part of the information needed to value a stand of timber because the price of various timber products differs greatly.

Generally, timber products are named based on their use: pulpwood, chip-n-saw, sawtimber, plywood logs, veneer logs, poletimber, etc. Timber product specifications vary based on factors such as final product specifications, mill technology, local timber supply and demand, among other factors. Within limits, specifications fluctuate in a given area largely based on supply and demand. Timber supply is strongly influenced by weather conditions. There are, however, rules of thumb for timber product specifications, as noted in the table below.

Product	Min 2" DBH Class	Top Diameter
Pulpwood	6"	2"
Chip-n-saw	8"	4"
Sawtimber	12"	8"

Specifications for other products such as pine super-pulpwood, plywood logs, and poles vary too much to allow for useful generalizations, or these products are not traded in all markets across the southeast.

Timber markets are decentralized and, in general, timber sale information is not publicly available, so Timber Mart-South (TMS) surveys professionals to produce and publish market price averages for both stumpage and delivered timber across 11 southeastern states. TMS published prices are based on actual

timber sale data as reported by numerous industry professionals across the region.

Foresters traditionally convert field measurements of trees to solid wood volume. However, timber arriving at a mill is now almost universally sold by weight. Conversion of solid wood volume to weight is based on wood density by tree species, as well as log rule conversions at common tree diameters (see Appendix D).

The Timber Mart-South south-wide average timber (stumpage) prices, 1<sup>st</sup> quarter 2017 are:

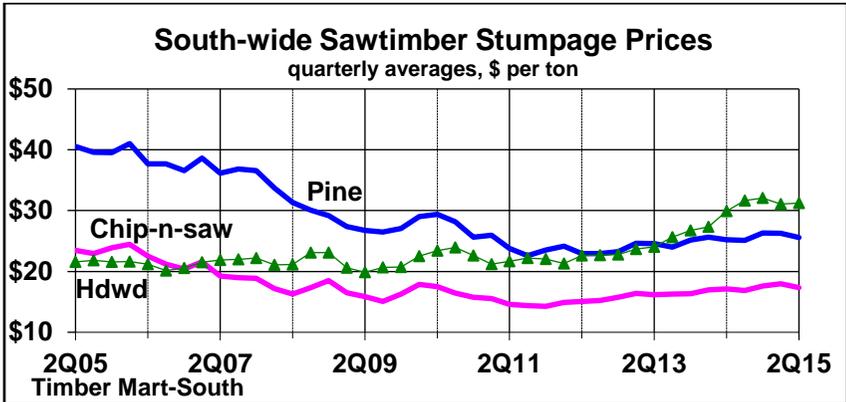
Product	\$ / ton
Pine sawtimber	23.75
Oak sawtimber	45.05
Mixed hardwood sawtimber	31.53
Pine chip-n-saw	16.99
Pine plywood logs	28.85
Pine poles	46.05
Pine pulpwood	9.57
Hardwood pulpwood	7.92

*Stumpage* is the price paid to the landowner for the right to harvest their timber. Loggers will fell trees and sort timber into product classes, then load the logs onto a truck and haul the timber products to a mill.

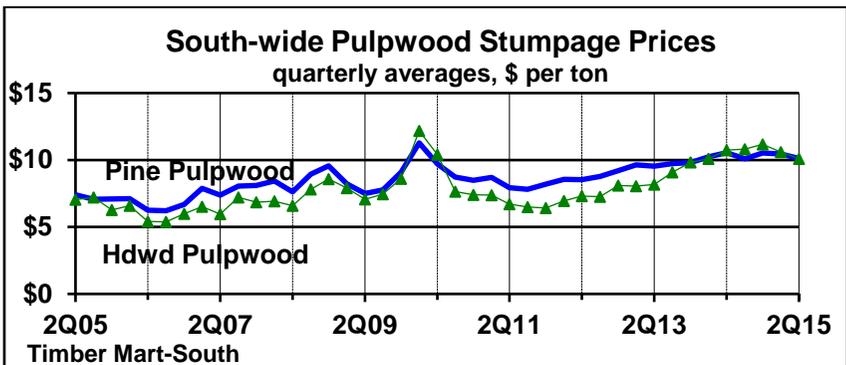
Mills pay a *delivered price* for the incoming raw material.

**Example** Using 1<sup>st</sup> Quarter 2017 south-wide prices, a mill might pay a supplier \$43.33 per ton for pine sawtimber delivered to the mill. The difference (\$19.58 per ton) between this delivered price and the stumpage paid to a landowner (\$23.75 per ton) would be used to cover costs to procure the timber and pay a logger to harvest, sort, and haul the wood to the mill.

Traditionally, south-wide average pine sawtimber stumpage prices have been greater than hardwood sawtimber. Within the past two years, however, they have converged and hardwood prices have surpassed pine.



South-wide average pine and hardwood pulpwood prices have trended similarly over the past ten years.



Regional prices will vary based on factors such as local and global market conditions, distance to mills, types of mills and product specification requirements, and access to transportation infrastructure, such as highways and ports.

## 6.14 STAND AND STOCK TABLES

A *stand table* is simply a description of the number of trees per acre by diameter class. It can be presented either as a table (shown below) or as a graph (shown in Section 6.8).

DBH class (in.)	TPA
7	7
8	17
9	42
10	58
11	61
12	22
13	9
14	2

A *stock table* is a description of the volume or weight per acre by diameter class. It can be presented either as a table or a graph.

DBH class (in.)	Volume per acre (ft <sup>3</sup> )
7	40
8	139
9	476
10	881
11	1,211
12	559
13	287
14	79

If there are no trees in some intermediate DBH classes, these classes should not be omitted - show a value of 0.

## 6.15 DOWN WOOD VOLUME

Down wood are former standing trees that are now lying near, or on, the ground surface. For stream surveys, down wood is considered logs that are firmly embedded into stream banks (Frazier et al. 2005).

To measure the volume of a piece of down wood, two measurements are required: the length of the wood, and the mid-point diameter. If they are more accessible, the small-end diameter and large-end diameter can be averaged to obtain the mid-point diameter. The volume can be estimated using the following equation, as long as the diameter and length are represented by the same units.

$$volume = \pi \left( \frac{diameter}{2} \right)^2 length$$

Other assumptions are needed as well, including:

1. The decay class above which, and including, the logs that need to be measured. This relates to the soundness of the logs. Thomas (1979) describes one classification system:

	Decay class				
	I	II	III	IV	V
<i>Bark</i>	Intact	Mostly intact	Mostly intact	Absent	Absent
<i>Integrity</i>	Sound	Sapwood rotting	Heartwood sound	Heartwood rotten	None
<i>Branches</i>	All present	Larger twigs present	Larger branches present	Branch stubs present	Absent

2. The minimum diameter after which the down wood does not "count" (since it is too small to be of value for wildlife and fisheries).

Idol et al. (1999) discuss the measurement of down woody debris in oak-hickory forests.

**Example** You measure the size of a down log to be 10 feet long. The small end diameter of the log is 0.5 feet, and the large end diameter is 1.2 feet. Since the log lies on the ground, a DBH tape was not used to estimate these diameters, thus they are in 10<sup>ths</sup> of feet, rather than inches. How much cubic foot volume is contained in the log.

1. Convert the diameters from feet to inches: 6" and 14.4".
2. Calculate the average, or mid-point diameter: 10.2".
3. Determine square feet of a circle with a diameter of 10.2" using the basal area equation:  $0.005454 (10.2^2) = 0.5674 \text{ ft}^2$ .
4. Given that the length of the log is measured in feet, and the surface area of one end of a cylinder with a mid-point diameter of 10.2" is in square feet, multiplying these two together (feet  $\times$  feet<sup>2</sup>) will provide us with an estimate of the cubic feet in the log.

$$\text{volume} = 0.5674 \text{ ft}^2 (10 \text{ feet}) = 5.674 \text{ ft}^3$$

## 6.16 SITE QUALITY

A *site* refers to the vegetative conditions at a particular location. Water, soils, and nutrient availability, as well as local climatic components, such as temperatures and solar radiation are all considered *site parameters* that play an important role in defining growing conditions.

*Site quality* has a number of meanings, many of which are closely related to each other. These include the following:

1. Timber-Oriented: The maximum amount of volume that can be produced over a given amount of time.
2. Ecosystem-Oriented: Total annual productivity, including all plants, animals, bacteria, etc., generally expressed as the potential of a site for biomass production.
3. Generalist: The capacity of an area to produce vegetation, dependent on climate, water availability, soil type, slope, aspect, and other biological factors.

Site quality can be expressed qualitatively or quantitatively. A few qualitative descriptions include these examples:

*"Good upland sites have deep, moist, but well-drained soils, face north to east, and are concave, and often found at the base of hills along streams."*

*"Poor sites include ridges, hot, dry, south and west slopes, shallow and stony soils, convex slopes, poorly drained soils, heavy clay, soils with iron or clay pan, and compacted soils."*

Other qualitative descriptors include habitat types (based on climax vegetation), indicator species, understory vegetation, and water-nutrient availability. An example is a *phytosociological system* of classifying plant communities depending on various

plant compositions conditioned by the abovementioned site parameters.

Based on the above examples, different sites could further be grouped into *site productivity classes*, such as I, II, III, etc.

To describe a site quantitatively, you can use direct methods or indirect methods. Direct methods are those where you would collect all the accumulated biomass grown on a site over a specific period and weigh or otherwise measure it. Indirect methods are where you would develop an indicator (an estimate) of productivity from a sample of trees or other vegetation.

### Site index

*Site index* is the most practical, consistent, and useful indirect method of measuring site quality, particularly with respect to timber production. Site index is defined as:

The average height of a sample of dominant and codominant trees at some specified “index” (or base) age.

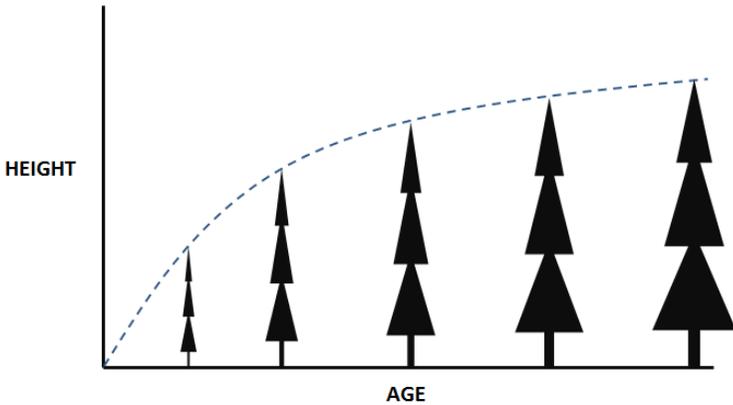
In recent decades the most commonly used sample observation of trees for site index determination is the *Total Height*, or *Top Height*. We would average a specified number of the largest diameter trees per given area (e.g., 100 largest trees per ha, or 40 per acre, or 1 largest tree per 100 m<sup>2</sup>) and use the average to estimate site index.

In the South, we generally use base age 25 for site indices.

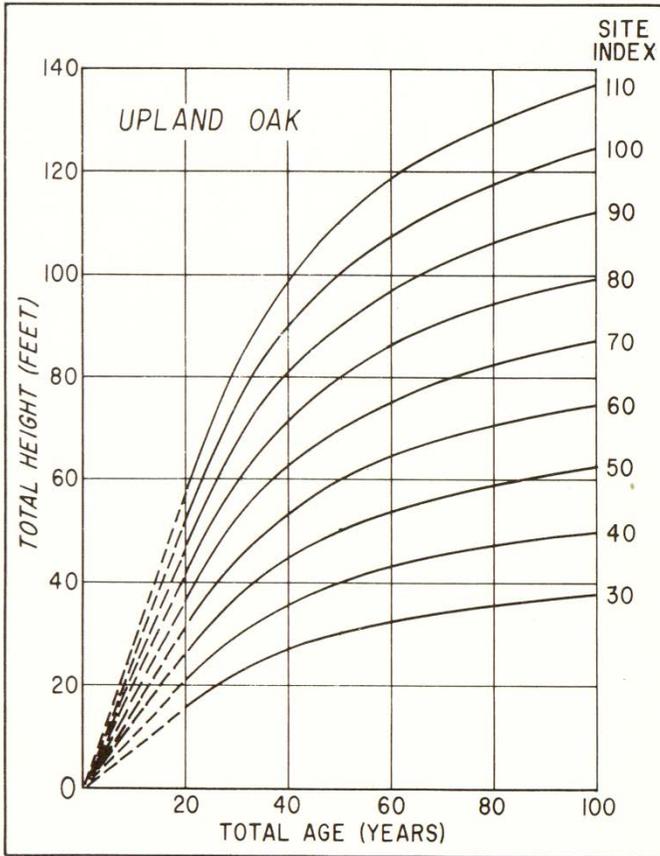
In the West, we generally use base age 50.

**Example** A site index 70 (base age 25) indicates that at age 25, we expect the dominant and codominant trees on a site to be 70 feet tall.

Dominant and codominant trees are used because they are assumed to be the trees that have been “free to grow” throughout their lifetime. As suggested, it is expected that the largest trees selected for total height determination are all in dominant or codominant positions in the forest canopy. Height development of these trees is relatively independent of the density of vegetation growing on the site within a broad range of densities.



Typically, growth curves are produced that represent dominant and codominant height development over time – these curves are known as *Site Index Curves*, *Site Index Height-Age Curves*, and even just *Site Curves*.



Source: USDA Forest Service Research Note No. SE-125.  
April 1959.

(Olson 1959)

### Important points

1. Trees located on higher site index areas will be taller, and consequently contain more volume, than trees located on lower site index areas.
2. A number of site index curves have been developed for specific conditions to reflect differences in the height development pattern of trees:

Natural stands vs. planted stands (plantations)

Different physiographic regions (Piedmont vs. Coastal Plain)  
Different species (longleaf pine, slash pine, loblolly pine)  
Different management factors

3. Site index curves come from site index equations. To develop a site index equation, you need to measure the total height and age of a representative sample of dominant and co-dominant trees over a certain period on permanent sample plots, or get the growth series data from stem analysis of an appropriate sample of fallen trees. To use a site index equation, you need the average age and average height of dominant and co-dominant trees

4. Site index is not an exact measure of productivity. The calculations contain error, as a result of subjective determination of dominant and codominant trees, and errors in height or age measurements. Therefore, we usually do not consider 2 or 3 feet difference in site index to be meaningful.

5. Site index varies across the landscape, due to slope position, aspect, and soil and water conditions.

6. Site index of a piece of land can be changed with certain management practices (fertilization, vegetation control, irrigation, genetics). Remember: SI represents the average height of the dominant and codominant trees at some base age - if you can make trees grow faster, they might be taller at that base age than under other circumstances.

## 6.17 STAND DENSITY

A number of measures are used to describe how much a site is being used by trees, and the intensity of competition between trees for the resources found at the site. High levels of stand density may imply that the growth rates of individual trees will slow down, because more trees are competing for limited resources. Low levels of stand density may imply that the resources found at a particular site are not being used efficiently.

Quantitative measures of stand density include:

- Trees per acre (TPA)
- Cords per acre
- Volume per acre
- Basal area per acre (BA)

Several of these concepts are described in more detail in other Chapters, where methods are provided to allow you to understand how they are calculated.

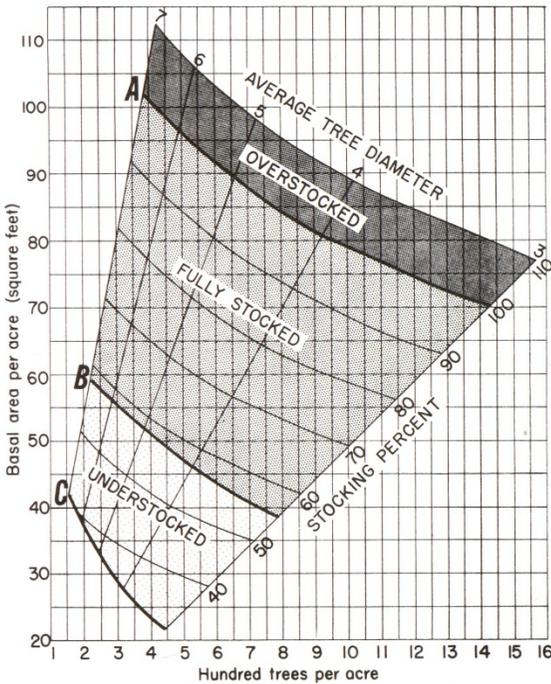
*Stocking* is a reference to how well the trees on a site meet some pre-defined management objective. Stocking can be expressed qualitatively (fully stocked, overstocked, understocked), or quantitatively (some percentage of full stocking).

Stocking is a relative concept. An overstocked stand from one management perspective may be an understocked stand from another management perspective.

Stocking guides allow you to understand how five measures of stand density relate to one another:

- Trees per acre
- Basal area per acre
- Average tree diameter
- Stocking percent (quantitative)
- Stocking (qualitative)

If you knew any two of the first four measures, you can estimate the others, and understand the qualitative measure of stocking.



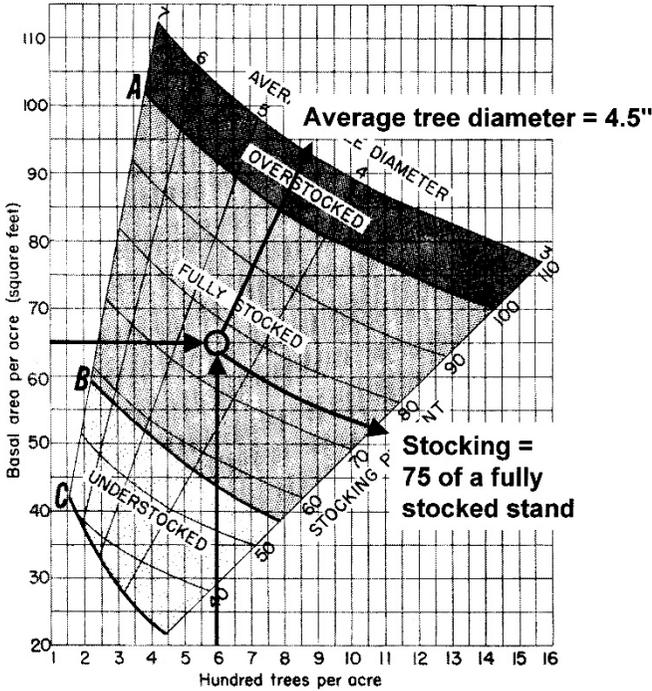
(Roach and Gingrich 1968)

**Example** Using the figure above for upland hardwoods, assume that a stand has 600 trees per acre, and 65 square feet of basal area per acre. What is your estimate of the average tree diameter, the stocking percent, and the qualitative stocking level?

First, find the intersection of 600 TPA (x-axis) and 65 square feet of basal area (y-axis).

Then extend a line parallel with the average diameter lines to the top of the stocking chart and estimate the average diameter (4.5"). Then extend a line to the right-hand side of the guide that is parallel with the stocking percent lines. The stocking is about 75 percent of a fully stocked stand.

Finally, notice that the intersection of 600 TPA and 65 ft<sup>2</sup> of basal area is within the "fully stocked" region of the chart. This is the qualitative measure of stocking.



**Example** Using the previous figure for upland hardwoods, assume that a stand has 500 trees per acre, and an average diameter of 4". What is your estimate of the basal area per acre, the stocking percent, and the qualitative stocking level?

BA = 45 ft<sup>2</sup> per acre

Stocking percent = 55%

This is an understocked stand.

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## 7.0 FIXED-AREA SAMPLING WITH PLOTS

In order to develop an estimate of the resources within an area that we manage, we must measure those resources. One way to accomplish this is with a system of fixed area plots (circles, squares, strips).

One could obviously measure all of the resources of interest within an area, but these 100% samples are time-consuming and expensive. Therefore, we generally sample an area, and develop estimates of resource values, along with associated measures of confidence in those estimates.

Here, we introduce a concept where we define a sample area in the field, and measure all of the vegetation within the area. Sampling is therefore based simply on whether vegetation is contained within the sampled areas.

This fixed-area sampling is frequently used to assess vegetation conditions in forested areas.

Wildlife and fisheries professionals also use this sampling method to understand the quantity and quality of woody vegetation in a resource management area.

After completing this Chapter, you will be able to:

1. Understand how one would use a fixed area plot to determine which trees to sample.
2. Calculate, from a sample collected within a fixed area, stand-level characteristics such as trees per acre.
3. Determine sampling intensity.
4. Develop a systematic sample of a resource based on a nominal (planned) sampling intensity.

## 7.1 THE FIXED AREAS

For forestry purposes, the most common fixed area plots are circular, and either 1/10 or 1/100 acre in size. One-tenth acre plots are commonly used for timber inventory purposes. One-one hundredth acre plots are commonly used for tree regeneration surveys. These plots are usually temporarily marked in the field.

Square plots are commonly used for research purposes, and their size varies. These plots are permanently marked in the field using corners established with re-bar or some other permanent fixture.

Rectangular strips have been used historically for timber inventory purposes. While they have been replaced by other sampling methods in forestry, strips continue to be used by wildlife professionals to measure down wood and other coarse woody debris.

Wildlife professionals and ecologists also use small (1 meter square) plots to assess the condition of grasses, forbs, and other vegetation. The use of these plots is beyond the scope of this manual.

### Important points

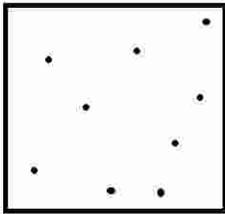
1. Trees or other resources in a stand can potentially be sampled if they fall within one of the fixed areas related to the sample. Generally, this implies that any one tree (or resource) can only be sampled once.
2. The choice to sample a tree (or other resource) is based on whether it falls within the fixed sample areas.
3. Each sampled tree (or other resource) represents the same, constant number of per unit area, no matter how large the tree (or resource) is (as compared to prism sampling).

## 7.2 SAMPLING SYSTEMS

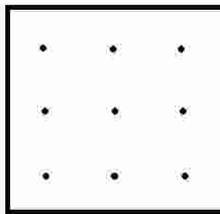
Two general types of sampling systems are used in association with fixed area plots: random and systematic.

Random sampling assumes that the location of each sampled area is randomly determined. In these cases, you would need to locate on a map the locations of the sampling areas, and base these locations on random numbers generated from a random number list. One way to do this is to arrange a map as a grid of X-Y coordinates, and generate random X-Y values in Excel to facilitate the location of the sampling areas. The drawback of this system is that it is somewhat difficult to locate and navigate to the random sample areas in the field.

Systematic sampling assumes that the location of the sampling areas is dependent on the location of the first sampling area, and the density of sampling areas required to meet the nominal (planned) sampling intensity. The advantage of this system is that you can navigate to each sampling area much easier than with randomly located plots, assuming your compass and pacing skills are adequate.



Random



Systematic

### 7.3 NOMINAL (PLANNED) SAMPLING INTENSITY

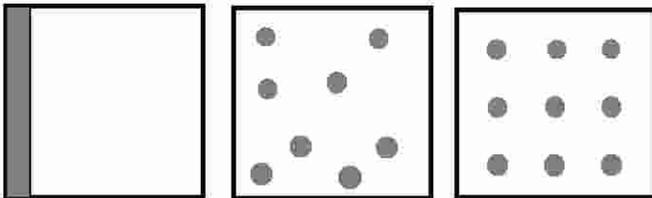
The nominal sampling intensity is simply how much of the area you plan to sample. If the sampling intensity were 100%, then you are indicating that you plan to sample everything. If the sampling intensity were 5%, then you are indicating that you plan to sample about 5% of the land or 5% (1 in 20) of the trees. We will develop a plan of action (a sample, or a cruise) based on the assumption that we plan to sample some amount of land and the resources found there. The formula to determining how much land area we plan to sample is:

$$\text{Area to be sampled} = \text{Stand size (acres)} \times \text{Sampling intensity (decimal percent)}$$

**Example** Assume that you need to sample the trees in a 120 acre stand, and that you decided to conduct a 5% sample. How much land area will you actually be sampling? 6 acres (120 acres  $\times$  0.05).

**Example** Assume that you need to sample the trees in a 20 acre stand, and that you decided to conduct a 10% sample. How much land area will you actually be sampling? 2 acres (20 acres  $\times$  0.10).

The question then, as it relates to these examples, is where to locate these sample areas.



Each of the above represent approximate 10% areas of a tract.

Let's assume from this point forward that we are interested in designing a systematic survey using fixed-area circular plots.

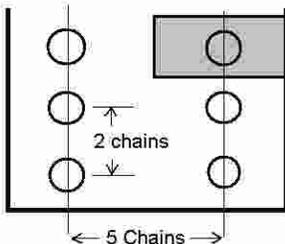
The sampling intensity that was assumed on the previous page can also be calculated if you had some information about the arrangement of the systematic survey.

Nominal sampling intensity can be calculated using the following formula:

$$\text{Sampling intensity} = \left( \frac{\text{Plot size (acres)}}{\text{Area each plot represents (acres)}} \right) 100$$

**Example** Assume that the arrangement of the sample survey is such that plots will be located using a grid that was 2 chains by 5 chains in size. Each plot can therefore be said to *represent* an area that was 2 chains by 5 chains in size, or 10 square chains, or 1 acre. If the sample plots were *actually* 1/10 of an acre in size, the sampling intensity would be

$$\text{Sampling intensity} = \left( \frac{0.10 \text{ acres}}{1 \text{ acre}} \right) 100 = 10\%$$



Recap: While the sample lines may be 5 chains apart, and the plots along each line 2 chains apart, the area each plot represents is also 5 chains by 2 chains (the shaded area above), even though the actual sampled area is the circular plot.

Sampling intensity will vary depending on a number of issues:

1. Variation within the forest. If there is a considerable amount of variation within a forest (e.g., a natural pine / hardwood stand), more samples, or a higher intensity sample, will be required to achieve a given level of precision. Less variation (e.g., a pine plantation) may require less samples, or a lower intensity sample.
2. Size of the stand. Larger stands of trees may require a lower intensity sample to achieve a given level of precision.
3. Value of the resource. The higher the value of the resource, the higher the intensity of the sample. In these cases, you want to be very comfortable with your estimate of value.
4. Sampling purpose. Some regeneration samples, habitat inventories, and young stand inventories may require a lower intensity than a sample designed for a purpose that involves an actual financial transaction (a timber sale).
5. Available time. If time is a limiting factor, a lower intensity sample may suffice.
6. Available budget. The sampling intensity should also be considered in conjunction with the funds available to perform the sample.

## 7.4 CIRCULAR PLOTS - WHAT TO SAMPLE

When using circular plots to sample trees, every tree within a radius around the plot center should be sampled.

"Every tree" includes those of the appropriate species and minimum DBH. These two parameters should be determined prior to starting the sample. For example,

Pine trees greater than 4" DBH  
Mast-producing oak trees greater than 6" DBH

To determine the appropriate plot radius, the following equation should be consulted:

$$\text{Area} = \pi r^2$$

Since we typically know the area of the plot, and we want to know the radius, we need to rearrange the equation:

$$\left( \frac{\text{Area}}{\pi} \right)^{1/2} = r$$

**Example** What is the radius of a 1/10 acre circular plot? There are 4,356 square feet in 1/10 of an acre, therefore the radius is

$$\left( \frac{4,356 \text{ feet}^2}{\pi} \right)^{1/2} = 37.2 \text{ feet}$$

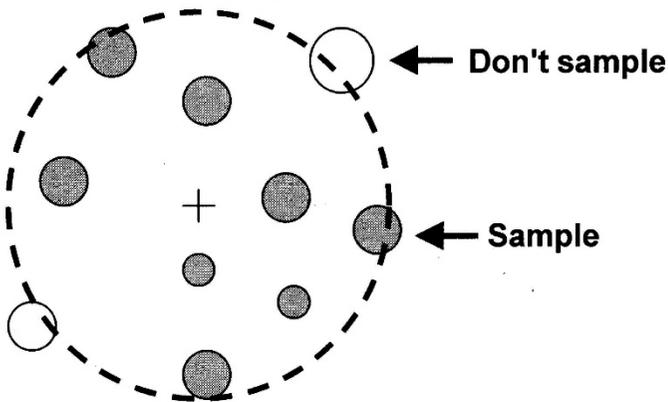
**Example** What is the radius of a 1/100 acre circular plot?

$$\left( \frac{435.6 \text{ feet}^2}{\pi} \right)^{1/2} = 11.8 \text{ feet}$$

Once the radius of the plot is known, the best way to implement this sampling system in the woods is for one person to hold a measuring tape at the plot center, and another person to measure out to each tree, to determine whether they fall within the plot radius.

This measurement is always made to the *center of the tree*, not the front, nor the back.

An overhead view of the plot radius and the trees that should and should not be sampled is shown below. The trees shaded gray should be sampled, because the center of these trees fall inside the plot radius.



### Important point

1. Trees of any and all sizes will be measured as long as they are within the plot, and are within the specifications of the trees to be measured.

## 7.5 PLOT SIZES

The following table lists the plot sizes in English and metric units for a number of standard circular plots used to measure trees and other resources.

Area (acres)	English system radius (feet)	Metric system radius (meters)
1/1000	3.7	1.13
1/100	11.8	3.59
1/10	37.2	11.35
1/5	52.7	16.05
1/2	83.3	25.38
1	117.8	35.89

## 7.6 NUMBER OF PLOTS NEEDED

When designing a sampling system, you will need to understand the number of plots that will be needed, so that you can allocate enough time to measure them, or determine how much it will cost to measure them.

To estimate the number of plots that are needed, three pieces of information are required:

- The area of the stand or tract to be sampled
- The sampling intensity
- The size of the sample plots

The equation to estimate the number of plots needed is:

$$\text{Number of plots needed} = \left( \frac{\text{Area to be sampled (acres)}}{\text{Area of a plot (acres)}} \right)$$

**Example** You need to sample mast-producing oak trees on a 160 acre stand using 1/10 acre plots. You plan to sample 10% of the stand, therefore the area to be sampled is 16 acres (see Section 7.3 if you are unsure of this). How many plots are needed?

$$\text{Number of plots needed} = \left( \frac{16 \text{ acres}}{0.10 \text{ acres}} \right) = 160 \text{ plots}$$

We say that this is an "estimate" of the number of plots needed because when you design a sample for a stand with an irregular boundary, you may find that the actual number of plots that you can fit into the stand may be 1-2 more or less than the planned number of plots.

In any event, you should sample the actual number of plots that fit into the stand rather than the planned number of plots.

## 7.7 REPRESENTATIVE AREA

As we noted earlier, each plot will "represent" an area of a stand that is being sampled. This holds true for all samples other than a 100% (i.e., complete) sample.

In Section 7.3 we illustrated a sample where circular plots were arranged on sample lines that were spaced 5 chains apart. The plots were spaced 2 chains apart along each line. Even though each plot may be 1/10 of an acre in size, they *represent* a larger area - 1 acre (2 chains × 5 chains).

To determine the representative area of each plot in a sample, you will need two pieces of information:

- The area of the stand or tract to be sampled
- The number of the sample plots needed (planned)

The equation to determine the representative area is:

$$\text{Representative area (acres)} = \left( \frac{\text{Area of the stand (acres)}}{\text{Number of plots needed}} \right)$$

**Example** Assume that you need to sample mast-producing oak trees on a 160 acre stand, and have determine that you need 160 1/10 acre plots. How much area does each 1/10 acre plot represent?

$$\text{Representative area (acres)} = \left( \frac{160 \text{ acres}}{160 \text{ plots}} \right) = 1 \text{ acre per plot}$$

## 7.8 PLOT AND SAMPLE LINE SPACING

When designing a systematic line-plot sample, two pieces of information are needed to draw the plan on a map:

- Distance between the sample (cruise) lines
- Distance between the plots along each sample (cruise) line

Using the representative area of each plot, and an assumption that you will make, we can determine the plot spacing. The assumption that you need to make involves specifying one of the two measures noted above. For example, you may assume that the distance between the plots along each sample line will be 2 chains. With this information and the representative area, you can determine how far apart the sample lines need to be.

One of these two equations will be useful in determining the plot spacing:

$$\text{Distance between sample lines } (ch) = \left( \frac{\text{Representative area } (ch^2)}{\text{Distance between plots along a line } (ch)} \right)$$

$$\text{Distance between plots along a line } (ch) = \left( \frac{\text{Representative area } (ch^2)}{\text{Distance between sample lines } (ch)} \right)$$

**Example** Assume that the representative area of a plot in a sample was 2 acres. Assume also that you want the distance between the sample lines to be 5 chains. How far apart should the sample plots be spaced along each line?

$$\text{Distance between plots along a line } (ch) = \left( \frac{20 \text{ } ch^2}{5 \text{ } ch} \right) = 4 \text{ } ch$$

## 7.9 LAYING OUT A SAMPLE

With the tools provided in the previous sections, one can design, or lay out, a systematic sample on a map using fixed-area plots.

Assume you know the following:

Representative area = 2 acres

Distance between the sample lines = 5 chains

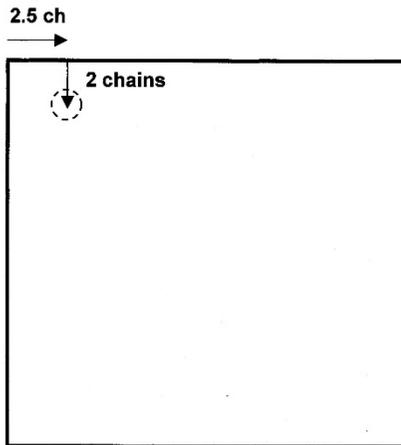
Distance between the plots along a sample line = 4 chains

If you had a rectangular tract that needed to be sampled, you would lay out the sample by beginning in one corner of the tract and by placing the center of the first plot....

1/2 of the distance assumed between the sample lines (2 ch)

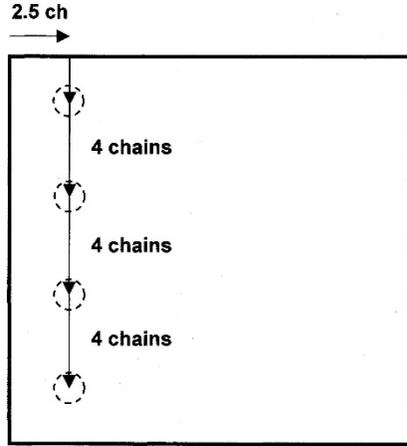
and

1/2 of the distance assumed between the plots along a sample line (2.5 ch).



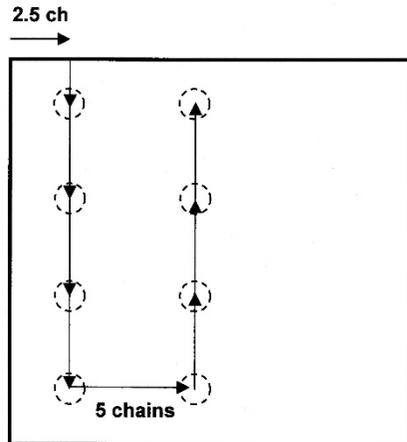
Assuming that the sample lines ran in a North-South direction, the next plot will be 4 chains south of the first plot, since the distance between plots along a sample line is 4 chains. This

continues until the plots for the entire sample line have been drawn.



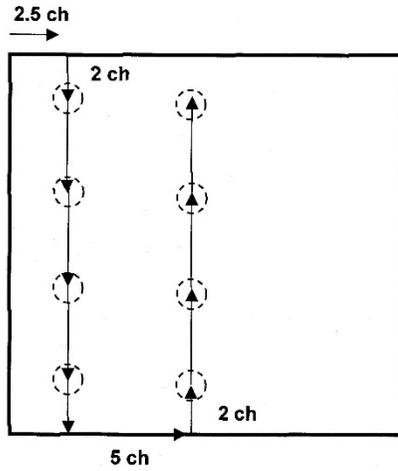
At this point, there are two options:

(1) Develop a perfectly rectangular grid of plots by moving East from the last plot 5 chains (the distance between sample lines) and developing a new line of plots that run South-North.

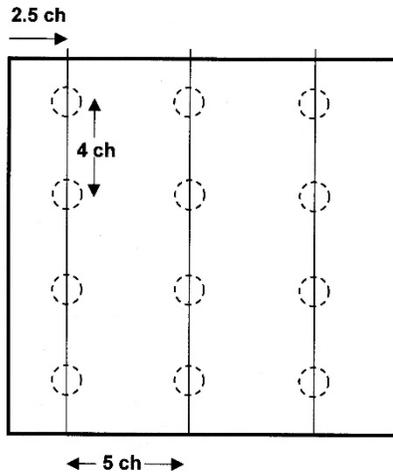


(2) Continue straight to the boundary of the tract along the first North-South sample line, then travel 5 chains East, then begin the next line 2 chains from the edge of the boundary (1/2 of the distance between plots along a sample line). This system may not

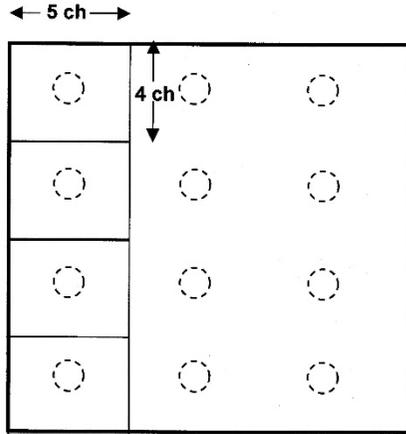
result in a perfectly rectangular grid of plots, however.



The remaining sample lines and plots are developed in a similar manner. For example, if you used option #1, your sample should look like this:



If you used option #1 (perfectly rectangular grid), by drawing lines between the plots, you should be able to subdivide the tract into rectangles that are exactly the size of the representative area.



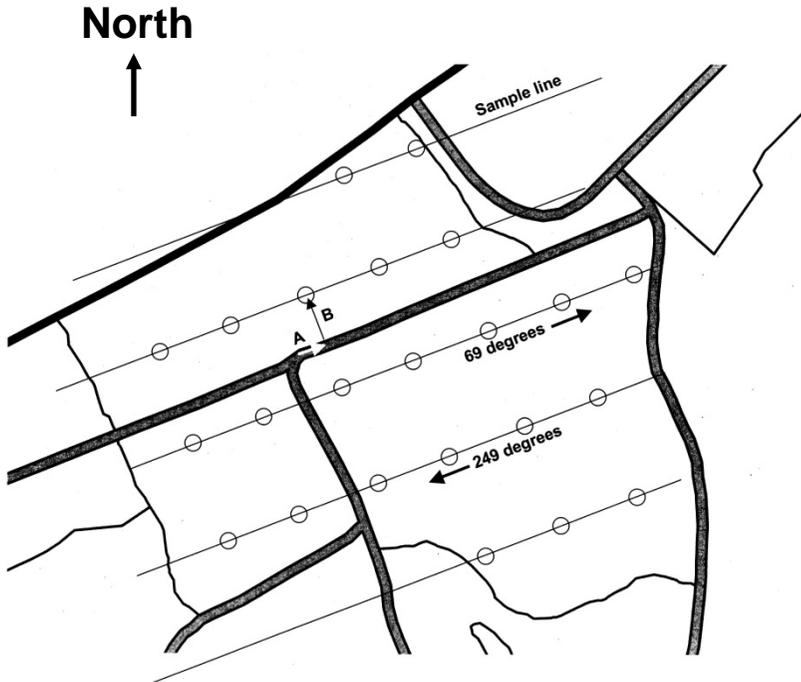
Unfortunately, most stands of trees in a forest are not perfect rectangles. Therefore, you will need to use your judgement to determine:

- (a) Which part (or corner) of the stand to begin the sample
- (b) How you want the sample lines oriented (North-South, East-West, or some other set of angles)

**Example** In the example shown on the next page, the sample lines are oriented parallel to a road.

One of the plots can be located by pacing  $1/2$  the normal distance between the plots (A) from a road intersection on an azimuth of  $69^\circ$ , then by pacing  $1/2$  of the normal distance between the sample lines (B) on an azimuth of  $339^\circ$ . This second azimuth is at a right angle to the  $69^\circ$  azimuth.

All of the other plots are located on a rectangular grid (albeit rotated  $69^\circ$ ) using the first plot as a reference.



In this sample, 23 plots will be sampled in the stand.

### Important points

1. No matter how many plots are needed, given the calculations in Section 7.6, if more (or less) can be laid out in a stand given the specifications of the sample (distances between plots and between lines), all of the plots must be sampled.
2. As much of the variation in trees should be sampled, where possible.
3. A boundary line, road, or right-of-way can be used as a baseline when orienting the grid of plots.
4. Sample lines should be run across (perpendicular to) windrows, or across rows of trees, not with the rows.

5. Sample lines should run directly toward, and not parallel with, streams, where possible.
6. Edges of stands need to be sampled just like other parts of a stand.
7. If part of a plot lies across the boundary of a stand, either move the plot back so that the entire plot lies within the stand, or use the boundary overlap correction described on page 221 of the book *Forest Measurements* by Avery and Burkhart (2002).
8. A  $5 \times 2$  chain grid may minimize the amount of time spent walking between plots in a stand, however a  $3 \times 3$  chain or  $4 \times 4$  chain grid make sample line layout a little less troublesome.

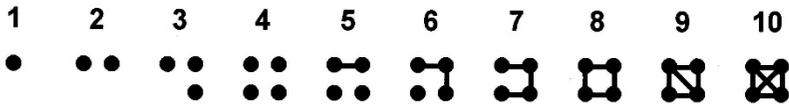
## 7.10 SAMPLING A PLOT

When you have navigated to a plot in a stand being sampled, you should do the following:

1. Make a note on your map (or hand-held data collector) of the current point number.
2. Mark the plot center with some flagging, or by scuffing the ground with your boot.
3. Pick a tree to begin the sample.
4. Work systematically around the point, clockwise or counter-clockwise, so that you do not miss any potential sample trees.
5. If using a field book, you might design a page to look like that shown below in Section 7.11.
6. Do not number the plots on the map before you sample them - number them on the map as you go.

### 7.11 DOT-DASH TALLY SYSTEM

One common method for tallying trees by diameter classes (as opposed to recording the exact DBH for each tree) is to record how many trees were sampled using the dot-dash tally system.



Here is an example from a field book, showing the number of trees sampled by 1-inch diameter classes for different height classes (one log = 16 feet).

38					
Stand 675		6/28/07			
PLOT Sampling (1/10 acre)		P. BETTINGER			
		B. FERNOW			
Plot # 1					
DBH	Height class (logs)				
class (in)	1.0	1.5	2.0	2.5	3.0
6		.			
7		.	..		
8			..	..	
9			..	..	.
10				..	.
11				..	..
12				..	..

## 7.12 CALCULATING TREES PER ACRE

Each tree measured in a fixed-area plot represents the same number of trees per acre, no matter how large it is. To determine how many trees per acre each tree represents, you need an expansion factor:

Expansion factor (EF) =  $1 / (\text{plot size (acres)})$

To determine the number of trees per acre from a single plot:

Multiply the number of trees measured on the plot (the tally) by the expansion factor.

$$\text{TPA} = (\text{EF}) (\text{Tally})$$

This estimate of TPA can be made for each tree as well, if different species or sizes of trees were measured on a plot.

**Example** You sampled 14 trees within a single 1/10 acre fixed area plot in a stand. What is the estimate of TPA for the stand?

$$\text{EF} = 10, \text{TPA} = 140$$

To determine the number of trees per acre from multiple plots:

a) Multiply the number of trees measured on each plot by the expansion factor, then average the estimates of TPA for each plot by dividing by the number of plots measured, or,

b) Multiply the total number of trees sampled (from all plots) by the expansion factor (assuming the same sized plot was used in all cases), then divide by the number of plots measured.

The first case (a) allows you to develop confidence intervals around the TPA, using TPA estimates for each plot as sample data. The second case does not allow you to do this.

### 7.13 CALCULATING BASAL AREA PER ACRE

Basal area is a measure of density in a stand of trees. It is the cross-sectional area of trees when measured at breast height (4.5 feet). Each sampled tree represents a different amount of basal area per acre. To compute basal area per acre, you need a two measurements:

- DBH of the tree
- Expansion factor

You can either calculate an estimate of basal area per tree,

$$\text{BA per tree (square feet)} = 0.005454 \text{ DBH}^2$$

or an estimate of the basal area per acre:

$$\text{BA per acre (square feet)} = \left( \frac{\sum (0.005454 \text{ DBH}^2 \text{ EF})}{\text{number of plots}} \right)$$

**Example** You sampled a 12.3" DBH slash pine tree. What is the estimate of the tree's basal area?

$$\text{BA} = 0.005454 (12.3)^2 = 0.825 \text{ square feet}$$

**Example** You sampled the following 12 trees on a 1/10 acre plot:

Tree	DBH (in.)
1	9.7
2	12.3
3	14.5
4	13.8
5	15.7
6	8.9
7	11.5
8	10.1
9	10.6
10	6.8
11	16.4
12	14.7

What is the estimate of basal area per acre?

Tree	DBH (in.)	BA per tree (ft <sup>2</sup> )	EF	BA per acre (ft <sup>2</sup> )
1	9.7	0.513	10	5.13
2	12.3	0.825	10	8.25
3	14.5	1.147	10	11.47
4	13.8	1.039	10	10.39
5	15.7	1.344	10	13.44
6	8.9	0.432	10	4.32
7	11.5	0.721	10	7.21
8	10.1	0.556	10	5.56
9	10.6	0.613	10	6.13
10	6.8	0.252	10	2.52
11	16.4	1.467	10	14.67
12	14.7	1.179	10	11.79
Total				100.88



### 7.14 CALCULATING VOLUME PER ACRE - ONE PLOT

In order to calculate volume per plot, you will need to compute a volume for each sampled tree.

**Example** Assume that the trees noted below had the per-tree volumes noted in column 3. Using the expansion factor for this plot (assume it is 10), one can arrive at a volume per acre estimate for this plot.

Tree	DBH (in.)	Tree volume (ft <sup>3</sup> )	EF	Vol / acre (ft <sup>3</sup> )
1	6.7	4.5	10	45
2	10.3	14.6	10	146
3	12.4	22.7	10	227
4	9.2	10.4	10	104
5	10.1	14.6	10	146
6	11.5	20.2	10	202
7	11.9	22.0	10	220
8	13.4	29.2	10	292

Total				1,382
-------	--	--	--	-------



## 7.15 CALCULATING VOLUME PER ACRE - MULTIPLE PLOTS

In many cases, when you are sampling a stand of trees using multiple sample plots, diameters are only recorded to the nearest one or two inch DBH class. While there is some perceived loss of precision by sampling diameters by diameter classes, the stand-level computations are much easier to perform. In the example below, data from 12 plots were combined into a single table by adding together all of the tallied trees for each DBH class. A single average height was assumed, and a single average tree volume for each DBH class was computed. The EF for each DBH class is the same (assume they were 1/10 acre plots). The volume per acre column is computed using the following equation:

$$\text{Volume per acre} = \left( \frac{(\text{Tally})(\text{Volume per tree})(\text{EF})}{\text{Number of plots}} \right)$$

**Example** Given the tally (tree count) from 12 plots in the first two columns of the table below, what is the volume per acre?

DBH class (in.)	Tally	Volume per tree (ft <sup>3</sup> )	EF	Vol / acre (ft <sup>3</sup> )
7	5	5.47	10	22.8
8	12	7.72	10	77.2
9	13	10.40	10	112.7
10	17	13.51	10	191.4
11	16	17.05	10	227.3
12	11	21.02	10	192.7
13	9	25.43	10	190.7
14	6	30.27	10	151.4
15	2	35.55	10	59.3

Total

1,225.5



Summary of the example in Section 7.15:

**DBH class:** These are the one-inch classes that were used to categorized sampled trees. When performing the sample, each tree must have been placed into one of these classes. For example, an 8.5" DBH tree falls into the 8" DBH class, as does a 7.6" tree.

**Tally:** These are the total number of trees sample across the 12 sample points. For example, the sum of the 13" trees on all 12 samples points was 9 trees.

**Volume per tree:** This is the average per-tree volume of each tree of a given DBH. Tree height was also used to create the volume, and an average tree height for each DBH class was used.

**EF:** This is the expansion factor associated with 1/10 acre plots.

**Volume per acre:** This is the tally times the volume per tree times the EF, divided by the number of sample plots (12). For the 15" DBH class, the calculation is:

$$\text{Volume per acre} = \left( \frac{(2)(35.55)(10)}{12} \right) = 59.3$$

The far-right volume per acre column of the table on the previous page can also be viewed as a *stock table* for the stand.

## **7.16 ADVANTAGES AND DISADVANTAGES OF FIXED AREA PLOT SAMPLING**

The advantages include the following:

1. Most of these processes are suitable for one-person sampling.
2. People performing the sample do not have to tally trees while also following a compass line. When using circular plots, a compass line is followed until arriving at the center of the plot, then sampling occurs.
3. People performing the sample pause at plot centers, allowing more time to check measurements.
4. People performing these samples are less hindered by brush and other obstacles than when sampling with strips.
5. The samples are separated by plots, therefore you can separate the plots into sets to calculate volumes for different timber types, etc. Statistical methods can be employed to determine variation, standard errors, and confidence intervals. As you will see with strip sampling, this is not the case.
6. The system is very useful for small stands, rights-of-way, or stands with small diameter trees.

The disadvantages include the following:

1. More time is required than when performing a prism sample.
2. More trees (and perhaps snags) are measured at each plot than when performing a prism sample.
3. The cumulative edge of the plots may be larger than the edges of strips in a strip sample, resulting in more borderline trees.

## **7.17 REFERENCES**

Avery, T.E., and H.E. Burkhart. 2002. *Forest Measurements*. McGraw-Hill, New York. 456 p.

## **8.0 FIXED-AREA SAMPLING WITH STRIPS**

Another fixed-area sampling method involves the use of strips rather than plots.

As we mentioned in the previous chapter, fixed-area sampling is frequently used to assess vegetation conditions in forested areas. Fixed-area sampling with strips was common in forestry 30 years ago, however it has been supplanted with the use of fixed area circular plots. Nonetheless, it is still used in a number of natural resource applications, such as to measure the amount of down wood in a stand of trees. Transects (lines) are also used to measure down wood, but we will not be covering that technique here. Transects are lines (not fixed areas), and any resource intersecting the transect is measured.

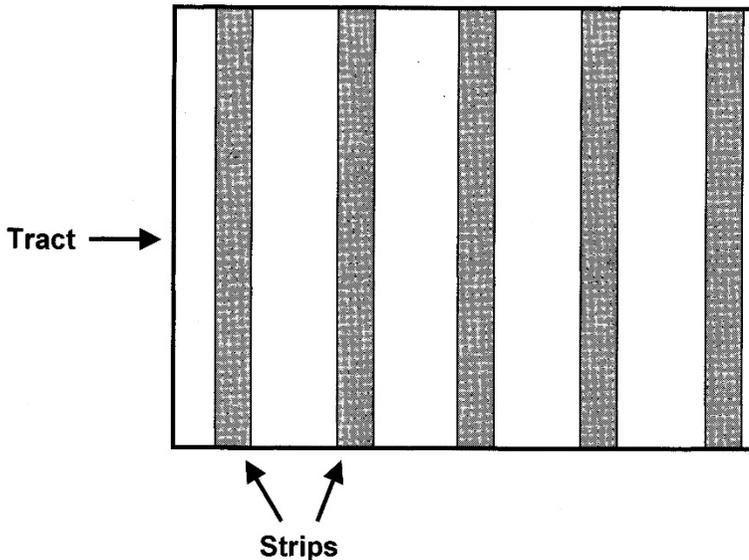
After completing this Chapter, you will be able to:

1. Understand how one would use a strip sample to collect information on forest resources within a pre-determined area.
2. Calculate, from a sample collected within a set of strips, stand-level characteristics such as the amount of down wood per acre.
3. Determine sampling intensity.
4. Develop a systematic sample of a resource based on a nominal (planned) sampling intensity.

## 8.1 THE FIXED AREAS

For forestry and wildlife purposes, the most common types of strips are rectangular in shape.

The most reasonable method for using strips is to also lay them out in a parallel manner on the landscape.



### Important conceptual points:

1. Trees (or other resources) in a stand can potentially be sampled if they fall within one of the fixed areas related to the sample. Generally, this implies that any one tree (or resource) can only be sampled once.
2. The choice to sample a tree (or other resource) is based on whether it falls within the fixed sample areas.
3. Each sampled tree (or other resource) represents the same number per unit area (as compared to prism sampling).

## 8.2 NOMINAL (PLANNED) SAMPLING INTENSITY

The nominal sampling intensity is simply how much of the area you plan to sample. If the sampling intensity were 100%, then you are indicating that you plan to sample everything. If the sampling intensity were 5%, then you are indicating that you plan to sample about 5% of the land, or 5% (1 in 20) of the trees. We will develop a plan of action (a sample, or a cruise) based on the assumption that we plan to sample some amount of land. The formula to determining the sampling intensity is:

$$\text{Intensity (\%)} = \left( \frac{W}{D} \right) 100$$

Where W = the width of the strip

D = the distance between the strips

Both W and D need to be stated in the same units.

**Example** Assume that you need to sample strips that are 1 chain wide, and are spread 10 chains apart. What is the nominal sampling intensity? 10%.

**Example** Assume that you need to sample strips that are 50 feet wide, and are spread 5 chains apart. What is the nominal sampling intensity? about 15%.

$$\text{Intensity (\%)} = \left( \frac{50 \text{ feet}}{330 \text{ feet}} \right) 100 = 15.2 \%$$

### 8.3 ACTUAL SAMPLING INTENSITY

The nominal sampling intensity represents the planned sampling intensity. Once the sample is laid out on a tract, the actual sampling intensity may be slightly different, depending on the shape of the tract, and how many strips can be placed inside the tract. The formula to determining the actual intensity is:

$$\text{Actual Intensity (\%)} = \left( \frac{\text{area in the sample}}{\text{total tract area}} \right) 100$$

Both the area in the sample and the total tract area need to be stated in the same units.

**Example** Assume that you sampled 10 strips, each was 10 chains long and 1 chain wide. Assume also that the size of the tract was 200 acres. What is the actual sampling intensity? 5%.

$$\begin{aligned} \text{area of a single strip} &= (\text{width}) (\text{length}) \\ &= (1 \text{ chain}) (10 \text{ chains}) \\ &= 10 \text{ sq. ch.} \\ &= 1 \text{ acre} \end{aligned}$$

$$\text{area in the sample} = (1 \text{ acre per strip}) (10 \text{ strips}) = 10 \text{ acres}$$

$$\text{total tract area} = 200 \text{ acres}$$

$$\text{Actual Intensity (\%)} = \left( \frac{10 \text{ acres}}{200 \text{ acres}} \right) 100 = 5\%$$

## 8.4 EXPANSION FACTOR

To develop tract-level estimates of resources, you will need to expand the estimate of resources found within the strips to the tract level using an expansion factor:

$$EF = \left( \frac{100}{\text{sampling intensity}} \right)$$

If you know the actual sampling intensity, use it here. If the tract area is unknown, use the planned sampling intensity. The sampling intensity is the percent (not the decimal percent).

**Example** Assume that you planned a sample for an area with an unknown size. The planned (nominal) sampling intensity is 10%. What is the expansion factor?

$$EF = \left( \frac{100}{10} \right) = 10$$

**Example** Assume that you sampled a 150 acre tract, using strips that were 1 chain wide. The total length of strips was 147 chains. What is the actual expansion factor?

Area in the strips = (1 ch) (147 ch) = 147 sq. ch. = 14.7 acres.

$$\text{Actual Intensity (\%)} = \left( \frac{14.7 \text{ acres}}{150 \text{ acres}} \right) 100 = 9.8\%$$

$$EF = \left( \frac{100}{9.8} \right) = 10.2$$

## 8.5 ESTIMATING TRACT AREA

If you did not know the size of the tract sampled, you can estimate it using the expansion factor and the dimensions of the strips.

Stand area (acres) = (acres sampled) (EF)

**Example** Assume that you planned a sample for an area with an unknown size. The planned (nominal) sampling intensity is 10%. If the length of strips was 135 chains, and the width of the strips was 1.0 chains, what is the estimated size of the tract?

$$\begin{aligned} \text{area sampled} &= (\text{length}) (\text{width}) \\ &= (135 \text{ ch}) (1 \text{ ch}) \\ &= 135 \text{ sq. ch.} \\ &= 13.5 \text{ acres} \end{aligned}$$

$$EF = \left( \frac{100}{10} \right) = 10$$

$$\begin{aligned} \text{Stand area (acres)} &= (\text{acres sampled}) (EF) \\ &= (13.5 \text{ acres}) (10) \\ &= 135 \text{ acres} \end{aligned}$$

## 8.6 STRIPS - WHAT TO SAMPLE

When using strips to sample trees, every tree (or down log in the case of coarse woody debris) within the strips should be sampled.

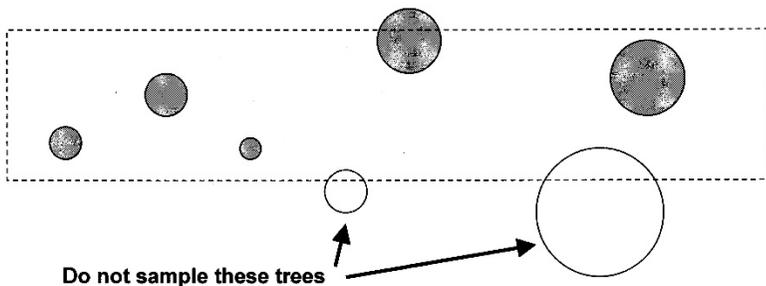
"Every tree" includes those of the appropriate species and minimum DBH. These two parameters should be determined prior to starting the sample. For example,

Pine trees greater than 4" DBH

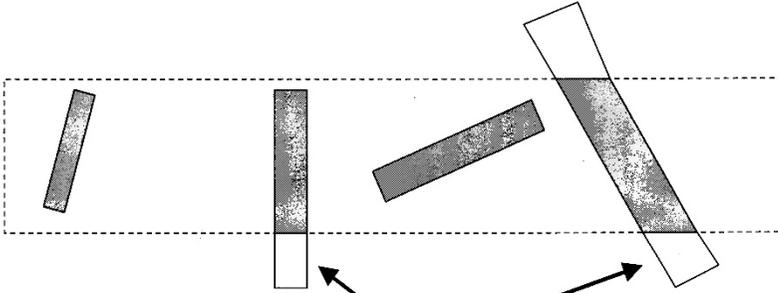
Mast-producing oak trees greater than 6" DBH

Once the strip width is known, the best way to implement this sampling system in the woods is for one person to navigate the strip centerline, and another person to measure out to each tree (1/2 of the strip width from the centerline), to determine whether they fall within the strip. This measurement is always made to the center of the tree, not the front, nor the back.

An overhead view of a strip and the trees that should and should not be sampled is shown below. The trees shaded gray should be sampled, because the center of these trees fall inside the strip.



When performing a strip sample for down logs, only the portions of each log that fall within a strip are measured. An overhead view of a strip and the trees that should and should not be sampled is shown on the next page.



**Do not sample these parts of down logs**

### Important points

1. A tree (or other resource) of any size will always be measured as long as it is within the plot, and is within the specifications of the trees (or other resource) to be measured.
2. Any sized down log will be measured as long as a portion of the log is within the plot, and the portion of the log within the strip meets the specifications of the down logs to be measured. The large end and small end diameters at the edge of the strip should be recorded, not the size of the ends of the log that lie well outside the strip.

## 8.7 LAYOUT OUT A SAMPLE

When designing a systematic sample for strips, a few pieces of information are needed to draw the plan on a map:

- Width of the strips
- Sampling intensity (nominal)

Assume that you needed to perform a 10% (nominal) sample using 1 chain wide strips. If you were to divide a tract into 1 chain rectangular areas (strips), every 10<sup>th</sup> strip would be sampled using a 10% sampling intensity.

For a 5% nominal sampling intensity, every 20<sup>th</sup> strip would be sampled. For a 20% nominal sampling intensity, every 5<sup>th</sup> strip would be sampled.

To lay out a strip sample plan on a map, begin by determining the distance between strip centerlines:

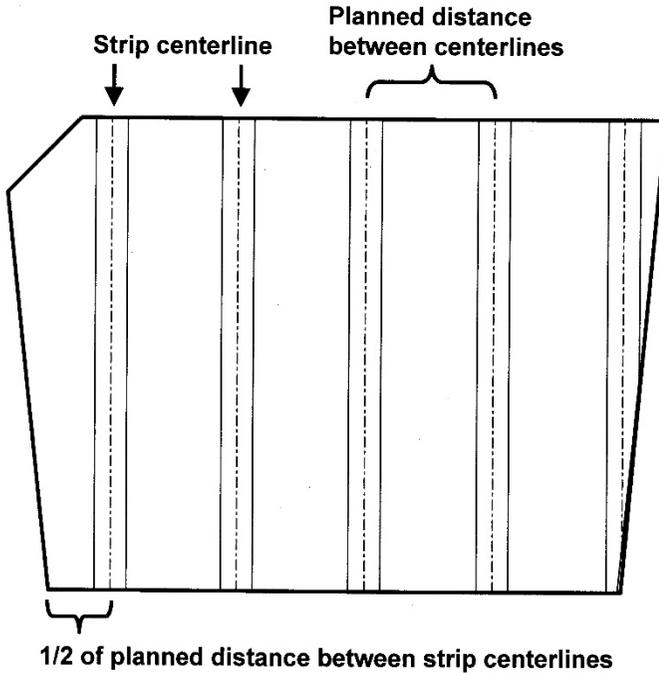
Distance between strip centerlines = (strip width) (EF)

**Example** Assume that a 10% nominal sampling intensity, and strip widths of 1 chain wide. What is the planned distance between the strip centerlines?

Distance between strip centerlines = (1 ch) (10) = 10 ch

To lay out the planned strip sample on a map,

1. Pick an edge of the tract to begin the mapped plan
2. Assume an orientation of the strips
3. Begin the centerline of the first strip 1/2 of the planned distance between the strip centerlines
4. Design the other strips so that they are spaced the planned distance between centerlines apart, and oriented in the same direction as the initial strip



### Important point

1. No matter how many strips are needed, given the calculations and assumptions used, if more (or less) can be laid out in a tract given the specifications of the sample, all of them must be sampled.

## **8.8 SAMPLING A STRIP**

When you have navigated to a strip in a stand being sampled, you should do the following:

1. Make a note on your map (or hand-held data collector) of the beginning of the strip.
2. Mark the strip center with some flagging.
3. Make a note of the edge of the strip in the direction to which you will be sampling.
4. Work systematically through the strip, so that you do not miss any potential sample trees.

## 8.9 CALCULATING RESOURCES PER ACRE

Each tree or down log measured in a fixed-area strip sample represents a different number per acre depending on the size of the strip. We will assume down logs were sampled in the next few examples.

### 1. Determine the number of down logs on the entire tract using the expansion factor:

Multiply the number of down logs measured in all of the strips by the expansion factor.

$$\text{Tract estimate of down logs} = (\text{EF}) (\text{Tally of down logs})$$

**Example** You sampled 114 down logs on 14 strips that were 1 chain wide in a 149 acre tract. The total strip length for all 14 strips was 143 chains. What is the estimate of the total down logs on the tract?

$$\begin{aligned} \text{Area sampled} &= (\text{width}) (\text{length}) \\ &= (1 \text{ chain}) (143 \text{ ch}) \\ &= 143 \text{ sq. ch.} \\ &= 14.3 \text{ acres} \end{aligned}$$

$$\text{Actual intensity (\%)} = \left( \frac{14.3 \text{ acres}}{149 \text{ acres}} \right) 100 = 9.6\%$$

$$EF = \left( \frac{100}{9.6} \right) = 10.42$$

$$\text{Estimate of total down logs} = (10.42) (114 \text{ logs}) = 1,188$$

2. Determine the number of down logs per acre:

To determine the number of down logs per acre, you need the size of the tract and the estimate of the number of down logs found within the tract.

Tract size = 149 acres

Estimate of total number of down logs = 1,188

Estimate of down logs per acre =

$$\left( \frac{1,188 \text{ down logs}}{149 \text{ acres}} \right) = 7.97 \text{ logs per acre}$$

## **8.10 ADVANTAGES AND DISADVANTAGES OF STRIP SAMPLING**

The advantages of using a strip sample include:

1. It is a simple system that is easy for an inexperienced person to learn and use.
2. Less time is wasted between sample areas than for another type of fixed plot sampling (e.g., circular plots) of equal intensity.
3. Fewer borderline trees are sampled, because the total perimeter of the sample area is smaller than with circular fixed plots.
4. There is less risk to personnel in remote areas: two people generally work together.

The disadvantages of using a strip sample include:

1. It requires at least two people to adequately perform the sample.
2. The edges of the strip are difficult to identify since the center of the strip is not always precisely defined. In addition, the edges must be located at a right angle to the strip centerline, which may be difficult to do unless a compass is used.
3. Since the sampled resources are contained within continuous strips, the data does not lend itself well to statistical analysis.
4. Inaccurate estimation of strip width may lead to errors.
5. It is difficult to maintain a straight strip centerline.
6. Brush and windfall are more of a hindrance in this system.
7. Spot-checking (auditing) the sample is difficult - the strip centerline is rarely exactly marked.

## 9.0 POINT SAMPLING

In the previous chapters the concepts of sampling within defined areas (plots, strips, etc.) were introduced. Here, we introduce a concept where only a sampling point is defined, and the vegetation sampled around the point is selected not based simply on the distance of the vegetation from the sampling point, but also on the size of vegetation.

While fixed area plot sampling is frequently used to assess vegetation conditions in forested areas, point sampling is used more often for timber cruising and inventory in the southern United States than plot sampling.

Wildlife and fisheries professionals also use this sampling method to understand the quantity and quality of woody vegetation in a resource management area.

After completing this Chapter, you will be able to:

1. Understand how one would use a prism to determine which trees to sample.
2. Understand the difference in basal area factors among prisms.
3. Calculate, from a sample collected with a prism, stand-level characteristics such as trees per acre.
4. Determine, if unknown, the basal area factor of a prism.

## 9.1 A BRIEF OVERVIEW OF POINT SAMPLING

Point sampling is a method of determining the appropriate trees to sample within a sampling or cruising system. The method is different from fixed-area methods in that a fixed area within which to sample is not assumed. The trees that will be sampled are chosen based on their size and their distance from each sample point, thus they can be very far away from a sample point and still be sampled. Point sampling has many names, such as:

Plotless sampling	Bitterlich sampling
Prism sampling	Prism cruising
Angle count sampling	Variable plot sampling

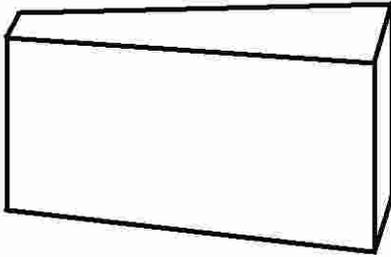
Each of these names describes a characteristic of the method - either the tool, the area sampled, or the developer of the method. For example, while Grosenbaugh (1952) may have introduced point sampling to America, it was Bitterlich (1948), an Austrian forester, who originally developed the process. Further, an angle gauge, or prism, is used to select the trees that will be sampled at each point, thus the "prism " and "angle count" references.

Point sampling is useful for determining basal area, volume, and trees per acre, along with other metrics commonly used in natural resource management and wood procurement. While the method may seem to favor forestry applications, point sampling is a simply a sampling method. Therefore, others have used it to collect forest structure data (e.g., basal area, and DBH) for such applications as breeding bird studies (Pojar 1995). The Wildlife Resources Management Manual developed by the U.S. Army Corps of Engineers (Mitchell et al. 1995) also suggests point sampling as a tool for estimating stand structure.

A number of angle gauges (e.g., Spiegel Relaskop) can be used, however we concentrate on the use of the glass wedge prism, because it is the most widely used device in the southern U.S.

## 9.2 THE WEDGE PRISM

A Prism is a small piece of glass that is tapered.



An angle is projected from the prism to each potential sample tree. The glass causes a distortion of light, displacing the image of the bole (in the prism) towards the thinner edge.

In other words, the bole of the tree is offset when you view it through a prism. This offset helps you determine whether or not to sample the tree.

### Important conceptual points:

1. All trees in a stand are potential sample trees at each prism point. This implies that a tree can be sampled more than once from more than one prism point.
2. The decision to sample a tree is based on each tree's size (DBH). Larger diameter trees have a higher probability of being sampled than smaller diameter trees, thus this method has a "probability proportional to size" sampling characteristic.
3. Each sampled tree represents a different number of trees per unit area (as compared to fixed area plot sampling).
4. Each sampled tree will represent the same basal area per acre, regardless of the tree's diameter.

### 9.3 HOLDING THE PRISM

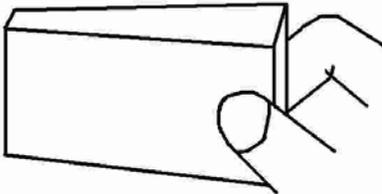
Assume that you have decided to use a specific sampling scheme: a systematic sample using a 2 chain by 5 chain sampling grid.

You will navigate from point to point using your compass and pacing skills.

Once you have arrived at one of the points in the grid, you should identify it in the field, by either tying some flagging to a piece of vegetation, or scuffing the ground with your foot. The main idea is to very clearly identify the point.

When you use a prism, you should hold it such that:

- a) The prism is at eye level, and is directed to a tree at a height that reflects the tree's DBH.
- b) The prism tapers away horizontally (tapers from left to right, or from right to left).



Held this way, a prism tapers from right to left.

- c) The prism is always over (on top of) the point location.

One common mistake that will bias your sample is to stand on the point, and hold the prism away from your body. Not only is the prism *not over the sample point*, it is closer to the trees being measured than it should be, by perhaps 1 or 2 feet.

## 9.4 BASAL AREA FACTORS

Each prism has associated with it a Basal Area Factor, or BAF.

The BAF indicates the amount of basal area per acre that each sampled tree represents (regardless of the tree's size).

A BAF 10 prism, therefore, suggests that each sampled tree represents 10 square feet of basal area per acre.

What is important here is that no matter how small or large the sampled trees are, each represents only a pre-defined (in this case 10 square feet) amount of basal area per acre.

*Example* At a single sample point, you sample the following using a BAF 10 prism:

<u>Tree</u>	<u>DBH</u>	<u>Height</u>
1	10	56
2	11	58
3	9	48
4	7	46
5	12	60
6	11	57

What is your estimate of the basal area per acre? 60 square feet. Six trees were sampled, and each represents 10 square feet per acre.

Common BAF prism factors include 5, 10, 20, 30, and 40.

## 9.5 WHAT YOU SEE WITH A PRISM

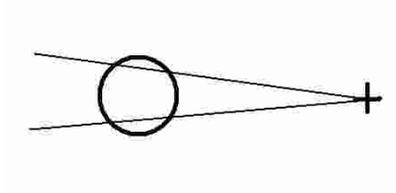
When held appropriately, you should see two things:

- The tree bole outside (above and below) the prism
- An offset portion of the tree bole inside the prism

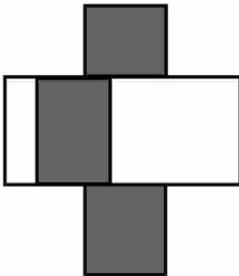
How much the tree bole is offset inside the prism (as compared to the bole outside the prism) will determine whether to sample the tree or not.

### A. A tree that needs to be sampled

The overhead view



What you see inside and outside of the prism

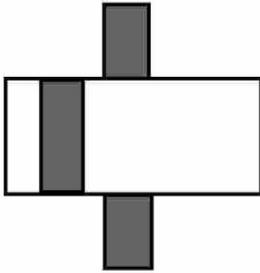


B. A tree that should not be sampled

The overhead view



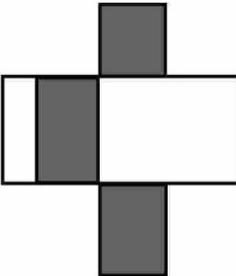
What you see inside and outside of the prism

C. A "borderline" tree (not sure whether to sample or not)

The overhead view



What you see inside and outside of the prism



## 9.6 BORDERLINE TREES

You may encounter one or more borderline trees at every prism point in your sample. Some have suggested counting these as half trees, or counting every other one encountered. Technically, there are no borderline trees if you have sufficient information on each. To determine exactly whether borderline trees should be measured, two additional measurements are needed:

- DBH
- Distance from the point to the center of the tree (see the overhead view below)



One common mistake is to measure from the prism point to the front edge of a tree, or to the back side of a tree. Always make this measurement to the center of the tree.

To determine whether to sample a tree, you need to use each prism's plot radius factor (PRF).

<u>Prism BAF</u>	<u>Prism PRF</u>
5	3.889
10	2.750
15	2.245
20	1.944
30	1.588
40	1.375

To determine whether to sample a borderline tree, multiply the DBH by the appropriate PRF. This is the critical distance. The

critical distance indicates the maximum distance that the tree can be away from the prism point, and still be considered a sample tree.

- ➔ If the tree is closer to the prism point than the critical distance, sample the tree.
- ➔ If the tree is farther away from the prism point than the critical distance, do not sample the tree.

**Example** You measured a borderline tree with a BAF 10 prism. The DBH of the tree is 10.1 inches, and the distance from the center of the tree to the prism point is 27 feet.

The critical distance is  $(10.1 \times 2.750) = 27.8$  feet

The distance to the tree was 27 feet.

- ➔ The tree is closer to the prism point than the critical distance, so measure (sample) the tree.

**Example** You measured a borderline tree with a BAF 20 prism. The DBH of the tree is 15.6 inches, and the distance from the center of the tree to the prism point is 31.6 feet.

The critical distance is  $(15.6 \times 1.944) = 30.3$  feet

The distance to the tree was 31.6 feet.

- ➔ The tree is farther from the prism point than the critical distance, so do not measure (sample) the tree.

### Important conceptual points:

1. Small DBH trees have to be close to a prism point to be sampled. A 1" DBH tree, when using a BAF 10 prism, can be, at most, 2.75 feet away from a sample point.
2. Large DBH trees can be far away from a point and still be sampled. A 30" DBH tree, when using a BAF 10 prism, can be up to 82.5 feet from a sample point and still be sampled.

## 9.7 SAMPLING A POINT

When you have navigated to a point in a stand being sampled, you should do the following:

1. Make a note on your map (or hand-held data collector) of the current point number.
2. Mark the point with an instrument (such as a stick or staff), some flagging, or by scuffing the ground with your boot.
3. Pick a tree to begin the sample.
4. Work systematically around the point, clockwise or counter-clockwise, so that you do not miss any potential sample trees.
5. If using a field book, you might design a page to look like that shown below.

36			
stand 345			5/1/07
Prism Sampling (BAF 10)			P. Bettinger
			G. Pinchot
Point #1			
	DBH		Height
Tree	(in)	Species	(ft)
1	6.7	LOB	37
2	10.3	LOB	53
3	12.4	slash	57
4	9.2	LOB	47
5	10.1	LOB	55
6	11.5	LOB	59
7	11.9	slash	60
8	13.4	LOB	63

## 9.8 CALCULATING TREES PER ACRE

Contrary to what you learned with fixed-area plot sampling, each tree measured in a prism sample does not represent the same number of trees per acre. To determine how many trees per acre each tree represents, you need two values:

- The BAF of the prism
- Basal area of the tree

With these two values, you can calculate each tree's Tree Factor (TF):

$$TF = (BAF / \text{Basal area of the tree})$$

**Example** You sampled a 10" DBH tree using a BAF 10 prism. What is the TF for the 10" tree?

$$\begin{aligned} DBH &= 10 \\ BA &= 0.5454 \\ TF &= 18.34 \end{aligned}$$

This 10" tree is then said to represent 18.34 trees per acre.

**Example** You sampled a 5.6" DBH tree using a BAF 10 prism. What is the TF for the 10" tree?

$$\begin{aligned} DBH &= 5.6 \\ BA &= 0.1710 \\ TF &= 58.47 \end{aligned}$$

This 5.6" tree then represents 58.47 trees per acre.

If both of these trees were sampled on a single prism point, one would represent 18.34 TPA, and the other would represent 58.47 TPA.

## 9.9 CALCULATING BASAL AREA PER ACRE

As we noted earlier, each sampled tree represents the same amount of basal area per acre when performing a prism sample - no matter how big the tree. To compute basal area, you need three measurements:

- The total tally
- The total number of points sampled
- The BAF of the prism

With these three values, you can calculate an estimate of basal area:

$$BA = \left( \frac{\textit{Tally}}{\text{Points sampled}} \right) \text{Prism BAF}$$

**Example** You sampled 8 trees on a point, and they are noted in the example in Section 9.7. What is your estimate of basal area per acre?

$$\text{Tally} = 8$$

$$\text{Points sampled} = 1$$

$$\text{BAF of the prism} = 10$$

$$\text{BA} = 80 \text{ square feet per acre}$$

**Example** You sampled 97 trees on 15 prism points, using a BAF 10 prism. What is your estimate of basal area per acre?

$$\text{Tally} = 97$$

$$\text{Points sampled} = 15$$

$$\text{BAF of the prism} = 10$$

$$\text{BA} = 64.7 \text{ square feet per acre}$$

### 9.10 CALCULATING VOLUME PER ACRE - ONE TREE, ONE POINT

In order to calculate volume per acre, you will need to compute a volume for each sampled tree, and the TF for each tree. The per-acre volume computation, or volume factor (VF) is then:

Volume per acre, or VF = (Tree volume) (TF)

The tree volume is simply the volume of a sampled tree. The TF indicates how many trees per acre that the one tree represents.

**Example** Assume you sampled a 19" DBH tree using a BAF 10 prism. Assume also that the tree has a volume of 64 cubic feet of wood. The TF associated with a 19" tree is 5.079. The volume per acre that this one tree represents is then:

Volume per acre =  $(64 \text{ ft}^3) (5.079) = 325.1 \text{ ft}^3$

**Example** Assume that the trees noted in Section 9.7 had the per-tree volumes noted in column 3 of the table below. Using the volume and TF for each tree, one can arrive at a volume per acre estimate for this point.

Tree	DBH (in.)	Volume (ft <sup>3</sup> )	TF	Vol / acre (ft <sup>3</sup> )
1	6.7	4.5	40.84	183.8
2	10.3	14.6	17.28	252.3
3	12.4	22.7	11.92	270.7
4	9.2	10.4	21.66	225.3
5	10.1	14.6	17.97	262.4
6	11.5	20.2	13.86	280.1
7	11.9	22.0	12.95	284.8
8	13.4	29.2	10.21	298.2
Total				2,057.6 ←

### 9.11 CALCULATING VOLUME PER ACRE - MULTIPLE POINTS

In many cases, when you are sampling a stand of trees using multiple prism points, diameters are only recorded to the nearest one or two inch DBH class. While there is some perceived loss of precision by sampling diameters by diameter classes, the stand-level computations are much easier to perform. In the example below, data from 12 points were combined into a single table by adding together all of the tallied trees for each DBH class. A single average height was assumed, and a single average volume was computed. The TF for each DBH class was determined and used to create an estimate of the volume per acre. The only difference between this example and the previous example is that the volume per acre column is computed using the equation:

$$Volume\ per\ acre = \left( \frac{(Tally)(VF)}{\text{Number of prism points}} \right)$$

**Example** Given the tally below from 12 prism points, what is the volume per acre?

DBH class (in.)	Tally	Volume per tree (ft <sup>3</sup> )	TF	VF	Vol / acre (ft <sup>3</sup> )
7	5	5.47	37.4	204.7	85.3
8	12	7.72	28.6	221.2	221.2
9	13	10.40	22.6	235.4	255.0
10	17	13.51	18.3	247.7	350.9
11	16	17.05	15.2	258.3	344.5
12	11	21.02	12.7	267.7	245.4
13	9	25.43	10.8	275.9	206.9
14	6	30.27	9.4	283.2	141.6
15	2	35.55	8.1	289.7	48.3
<b>Total</b>					<b>1,889.1</b> ←

Summary of the example in Section 9.11:

**DBH class:** These are the one-inch classes that were used to categorized sampled trees. When performing the sample in the woods, each tree must have been placed into one of these classes. For example, an 8.5" DBH tree falls into the 8" DBH class, as does a 7.6" tree.

**Tally:** These are the total number of trees sample across the 12 sample points. For example, the sum of the 13" trees on all 12 samples points was 9 trees.

**Volume per tree:** This is the average per-tree volume of each tree of a given DBH. Tree height was also used to create the volume, and an average tree height for each DBH class was used.

**TF:** This is the prism BAF divided by the basal area of a single tree of a given DBH. For example, for a 10" tree, the TF is:

$$TF = (10 / 0.5454)$$

TF indicates how many trees per acre each tallied tree will represent in the stand-level computations.

**VF:** TF times volume per tree, as noted in Section 9.10. This indicates the amount of volume each tallied tree represents. Rounding within the spreadsheet causes some error in the table.

**Volume per acre:** This is the tally times the VF, divided by the number of prism points (12). For the 15" DBH class it is:

$$Volume\ per\ acre = \left( \frac{(2)(289.7)}{12} \right) = 48.3$$

The far-right volume per acre column can also be viewed as a stock table for the stand.

## 9.12 DETERMINING THE BAF OF AN UNKNOWN PRISM

Most prisms have some type of mark on them to indicate their BAF. However, if not etched into the glass, these marks can eventually rub off. If you do not know the BAF of a prism, a simple formula can be employed to estimate it. Two measurements from a single sample tree are needed:

- Tree DBH
- Distance to the tree, where it becomes "borderline"

To obtain these measurements, first select a tree and measure its DBH. Then, backing away from the tree, note the point, when viewing the tree through the unknown prism, where it becomes borderline. Measure this distance, to the center of the tree.

Before using the formula noted below, convert the tree DBH to tree width in feet, by dividing the DBH by 12 (inches per foot).

$$BAF = 10,980 \left( \frac{\text{Tree width } (ft)}{\text{Distance to tree } (ft)} \right)^2$$

**Example** You found a prism in a drawer in the field office where you work, and need to know the BAF before you can begin sampling. You select a tree with a DBH of 25" (width = 2.0833 feet). Looking through the prism, you back away until the tree is "borderline," at 34.5 feet.

$$BAF = 10,980 \left( \frac{2.0833}{34.5} \right)^2 = 40.03$$

As a check on these computations, you might recall that the PRF for a BAF 40 prism was 1.375. Therefore, the critical distance is 34.4 feet ( $1.375 \times 25$ ), about where you thought that the 25" tree was "borderline."

### 9.13 CHOOSING A BAF

Ideally, when performing prism sampling, you want to use a prism that allows you to sample between 5 and 12 trees per point.

If you know ahead of time a "ballpark" average basal area per acre, you can work backward into the choice of a prism.

**Example** You are working in the southern U.S., and need to sample a stand of trees where the average basal area is about 100 square feet per acre (guessing). With a BAF 10 prism, you can expect to sample 10 trees at each point, on average (10 trees, each represents 10 square feet of basal area, provides a sample of 100 square feet of basal area). A BAF 5 prism will require you to sample about 20 trees per point. A BAF 20 prism will require you to sample about 5 trees at each point. Therefore, a BAF 10 or BAF 20 prism seems appropriate here.

**Example** You are working in the western U.S., and need to sample a stand of trees where the average basal area is about 230 square feet per acre (guessing). The number of trees you would need to sample at each point using different prisms is:

BAF 5 prism.....	46.00
BAF 10 prism.....	23.00
BAF 20 prism.....	11.50
BAF 40 prism.....	5.75

Therefore, you would select a BAF 20 or BAF 40 prism for this work.

## 9.14 ADVANTAGES AND DISADVANTAGES OF POINT SAMPLING

The advantages of point sampling include:

1. This method of sampling is much faster than systems that use fixed-area plots or strips.
2. Some computations, such as basal area, are much easier to obtain than when using fixed-area sampling.
3. One person can perform the sampling method.
4. More points, thus perhaps more area, can be covered in one day than the number of plots in a fixed-area plot sample.
5. Plot boundaries do not have to be identified.
6. The sampling effort is concentrated on larger, and potentially more valuable trees (for forestry and wildlife purposes).

The disadvantages of point sampling include:

1. Some computations, such as trees per unit area, are much more difficult to obtain than when using fixed-area sampling.
2. Some large sample trees may be further away from the sample point than the radius of a typical 1/10 acre fixed-area plot (37.2 feet).
3. One of the largest sources of error when using this system involves not sampling certain trees (omitting trees from a sample) because visibility is obscured by other trees or brush.

## 9.15 FINAL THOUGHTS

1. If there is another tree between the point you are sampling and a tree of interest, move one or more steps to the side of the point, maintaining the same distance to the tree of interest, and determine with the prism whether it is in, or out, of the sample.
2. The distance between your eye and the prism is not important as long as the tree can be seen clearly.
3. Prisms are frequently lost in the woods. They cost between \$25 and \$65 each.

### To prevent losing a prism in the woods:

Tie a bright piece of flagging to the lanyard (the piece of string or cord that is attached to the prism).

Using a brightly colored lanyard.

Tie the prism, using a light cord, to your cruising vest. This way, you can drop the prism while performing other measurements, and not lose it.

### To locate a lost prism in the woods:

Try to determine your last sample point.

Retrace your steps to the last sample point. (Being able to pace accurately and use your compass appropriately will enable you to return to the previous prism point.)

Search for the brightly colored lanyard or flagging. If these were not used, keep in mind that the prism may be hanging from a bush or shrub by its lanyard.

If no lanyard was used, search the ground thoroughly.

4. The fact that a single tree (perhaps a large tree) can be measured on more than one sample point should be accepted as normal practice. These trees should be considered valid sample trees and tallied as such.

## 9.16 REFERENCES

- Bitterlich, W. 1948. Die Winkelzählprobe. Allgemeine Forstund Holzwirtschaftliche Zeitung. 59(1/2): 4-5.
- Grosenbaugh, L.R. 1952. Plotless timber estimates – new, fast, easy. Journal of Forestry. 50: 32-37.
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- Pojar, R.A. 1995. Breeding bird communities in aspen forests of the sub-boreal spruce (dk subzone) in the Prince Rupert Forest Region. Province of British Columbia, Ministry of Forests Research Program, Land Management Handbook 33.

## 10.0 STATISTICS

Statistics are used to help us understand the world and make decisions at both the individual and population level. This is done by collecting, organizing, analyzing, and interpreting data – things that you will become familiar with during this course. To begin, we must understand the terminology frequently used in statistics.

*Population* – All individuals that we wish to know something about. For example, *all* of the pine trees in the coastal region of Georgia or *all* of the raccoons in Clarke County. We may want to know about the average age of students who are majoring in wildlife within the Warnell School of Forestry and Natural Resources – thus, the population is *all* individuals currently majoring in wildlife within our school. From these simple examples we see that a population can vary in size from small to large and, in fact, populations can be of any size.

The concept of a population is actually very simple:

- Populations can be constrained based on objectives
- Always remember to define the spatial and temporal characteristics of a population of interest.

If we are interested in determining the present volume of merchantable timber in the Oconee National Forest, what would our population be?

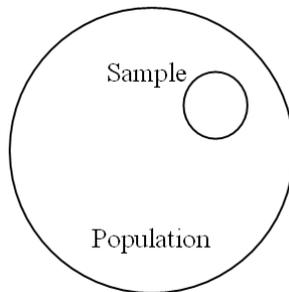
Answer: *All* of the trees that are of a harvestable size within the Oconee National Forest.

If we are interested in the people that visit Old Faithful Geyser at Yellowstone NP, how many potential populations are there?

Answer: Many (depending on our objectives). It could be *all* of the international visitors at Old Faithful, *all* of the people that visit Old Faithful on Sundays, *all* of the people that visit in motorhomes, etc.

We rarely have the time and financial resources to collect information from every single individual in the population. Instead, we can collect information from a sample and make *inferences* (conclusions) about a population.

*Sample* – The portion / subset of the population that we actually collect data from. The key assumption is that the information obtained from the sample reliably represents the population, otherwise it may be a biased sample which can lead to incorrect assessment of the population.



If we are interested in estimating the volume of timber in a 50 acre stand (~1,800 trees), what are some potential ways we can sample?

Answer: Randomly select 100 trees (the sample), obtain an average volume per tree and then multiply that by 1,800 for a population estimate.

If we are interested in the opinions of people that visit Old Faithful on Sundays, what are some potential ways we could sample?

Answer: Approach every 5<sup>th</sup> visitor, sample every other hour, or sample a random Sunday per month.

Values that summarize properties of the population are called *parameters*. Common parameters are the population average ( $\mu$ ) and the population variance ( $\sigma^2$ ). A value that estimates the

property of a sample is called a *statistic*. Common statistics are the sample mean ( $\bar{x}$ ) and the sample variance ( $s^2$ ).

*Variable* – The measurable characteristic (including appropriate units) of the sample/population of interest (i.e., what we're measuring).

Example variables: (can be Quantitative or Qualitative)

- Age
- Length
- Density
- Species



Using the previous example, the 50 acre stand (1,800 trees) would be the *population*, the 100 trees would be the *sample*, and the volume of each of the 100 trees would be the *variable*.

*Observation* – The actual measurement of the variable we have obtained (including appropriate units).

Example observations:

- Tree height = 57 feet
- Deer age = 3 years
- Survey gender = female
- Density of adelgids = 2.4 per cm
- Species = *Pinus strobus*

It should be noted that since samples represent only a small part of any real world population, it is almost certain that our sample statistic will not equal the population parameter. Thus, a great deal of *statistical inference* (expanding conclusions based on the sample to the entire population from which the sample was drawn) has to do with determining how far a sample statistic may be from a population parameter. If we determine that there is a good chance that our sample statistic is *close* to the population parameter it will be more useful to us than if we determine that

our sample statistic may be *far* from the population parameter.

After completing this Chapter, you will be able to:

1. Understand the basics of statistical inference.
2. Understand the various types of data and associated data description methods.
3. Consider multiple ways to represent data graphically and numerically.
4. Calculate measures of central tendency and dispersion.
5. Determine the reliability of a sample estimate.
6. Comprehend hypothesis testing and statistical significance.

## 10.1 TYPES OF DATA

Data are generally categorized as *Qualitative* and *Quantitative*.

*Qualitative Data* – Data that are associated with descriptions or categories (words) and are typically observed rather than numerically measured. Examples of qualitative data include eye color, political party, opinion, and gender. Clearly, individuals can be placed into these categories, but they are not the result of measurements. Qualitative data are classified as:

Nominal – there is no natural order between the categories such as with eye color (brown eyes are no better than blue eyes)

Ordinal – an order or rank exists (such as with teacher evaluations – very true, mostly true, partly true, not true)

*Quantitative* – Data that result from measurements or counts (numbers). For example, the number of offspring, the diameter of a tree, or age. Quantitative data are classified as:

Continuous – the measurements can take any value within a range (such as measures of size (height, weight, volume, etc.) or time)

Discrete – the measurements are integers (no fractional component) such as counts (number of deer, number of trees, etc.) and ages

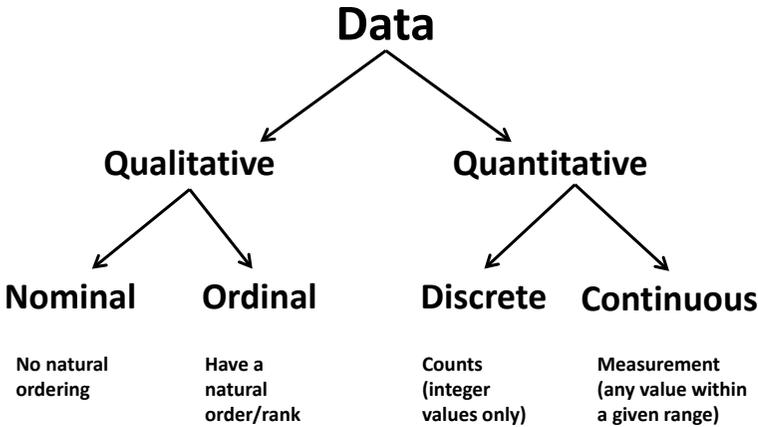
Keep in mind that information about a variable can be obtained as either type of data (*qualitative* or *quantitative*) depending on your objectives.

In what form might you record tree heights?

*Qualitative:* You could choose from a category (a suppressed, intermediate, co-dominant, or dominant tree).

*Quantitative:* You could physically measure the tree and determine its height (e.g., 65 feet).

To help you further visualize the differences, the following conceptual model might be of value.



Qualitative data (nominal and ordinal) are still considered qualitative data even when they are recorded with a numeric code. For example,

- 1 = disagree; 2 = no opinion; 3 = agree
- 1 = female; 2 = male
- 1 = healthy; 2 = moderate decline; 3 = severe decline

It is important to identify whether the data are quantitative or qualitative as this affects which type of description (graphs and tables) you can use as well as which statistics can be produced.

## 10.2 GRAPHICAL / TABULAR DATA DESCRIPTION

Data description means exactly what it implies – we need to describe our data to get an idea of what they ‘look like’. This will allow us to help make inferences about the population of interest.

There are two primary methods for describing data:

*Graphically* – with histograms, bar graphs, pie charts, scatter plots, etc.

*Numerically* – with tables that contain data such as frequencies, means, ranges, variability, etc.

### Types of data descriptions

- 1) Histograms – These depict the distribution of the data.
  - a) Frequency
  - b) Relative frequency
  - c) Cumulative frequency
  - d) Cumulative relative frequency

} Valid for quantitative or qualitative data

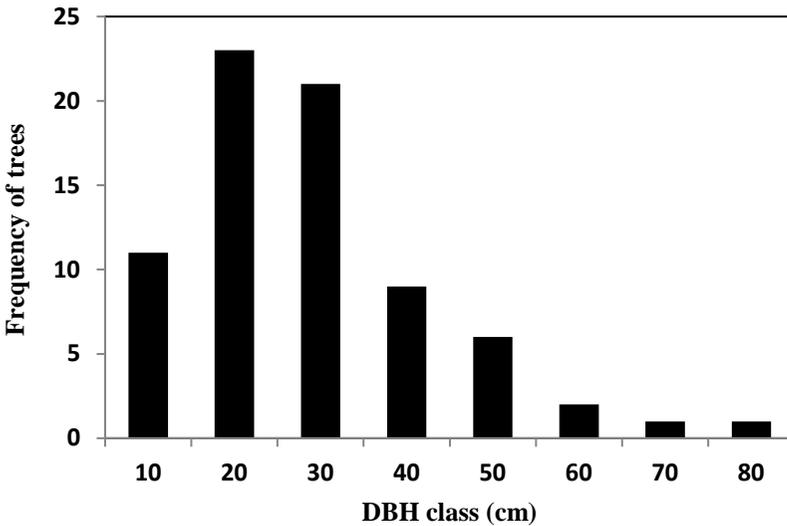
} Only valid for quantitative data
- 2) Ogives – These are linear versions of cumulative histograms.
- 3) Bar graphs – These are similar to frequency histograms, but usually associated with categories of qualitative data.
- 4) Pie chart – These are circular graphs showing how a total quantity is distributed among a group of categories (proportionate / relative frequency).
- 5) Line graph – These are used to display the change in one variable in relation to another (e.g., change over time).
- 6) X/Y chart / Scatter plot – These are used to display the change in one variable in relation to another, as well as the variation in the relationship between X and Y.

**Example** You are asked to measure the diameter (DBH) of a sample of trees ( $n = 74$  trees) in a specified area. Once you collect the data, you catalog the observations into DBH classes (e.g. DBH class 10 contains all trees with a DBH in the range 5-15 cm).

1a) Frequency histogram / table – Shows the number (frequency) of observations in each non-overlapping variable (DBH) class. The Y-axis will range from 1 –  $n$ .

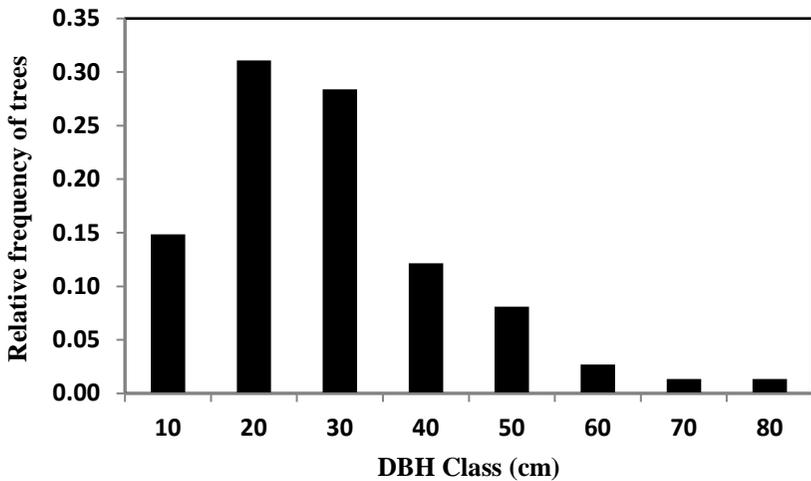
DBH class (cm)	Frequency
10	11
20	23
30	21
40	9
50	6
60	2
70	1
80	1
Total	74

Number of trees with a DBH that fall into that DBH class.



1b) Relative frequency histogram / table – Shows the fraction (proportion) of the total observations in each non-overlapping variable class (DBH). The Y-axis values will range from 0 – 1.

DBH class (cm)	Frequency	Relative frequency
←10 = 11 / 74	11	0.15
←20 = 23 / 74	23	0.31
←30 = 21 / 74	21	0.28
40	9	0.12
50	6	0.08
60	2	0.03
70	1	0.01
80	1	0.01
Total	74	1.00



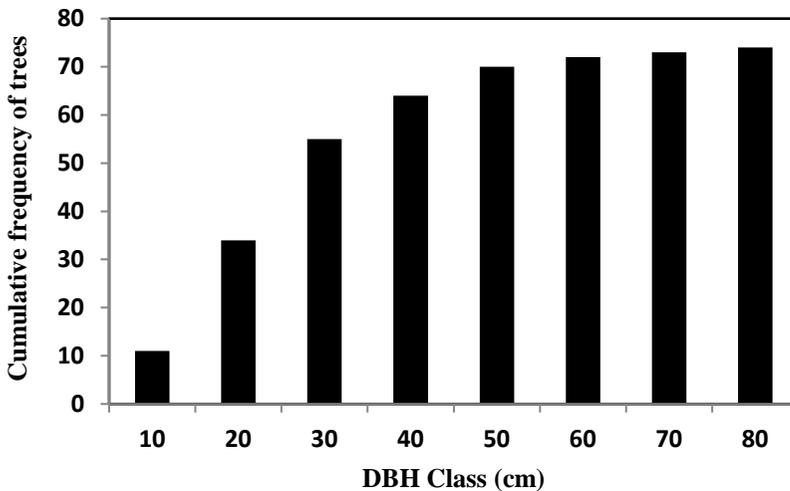
From afar, the frequency histogram and the relative frequency histogram look the same (only difference is the Y-axis), but the results are described differently. The value in the 20-cm DBH class would be described as:

Frequency histogram: “In the area sampled, there were 23 trees with a DBH in the 20-cm DBH class.”

Relative Frequency histogram: “In the area sampled, 31% of the trees had a DBH in the 20-cm DBH class.”

1c) Cumulative frequency histogram / table – Shows the frequency (number) of a variable class plus the frequencies of the classes below it (i.e., the number of observations that are less than or equal to the upper limit of each class). The Y-axis will range from 1 –  $n$ . This is only valid for Quantitative data.

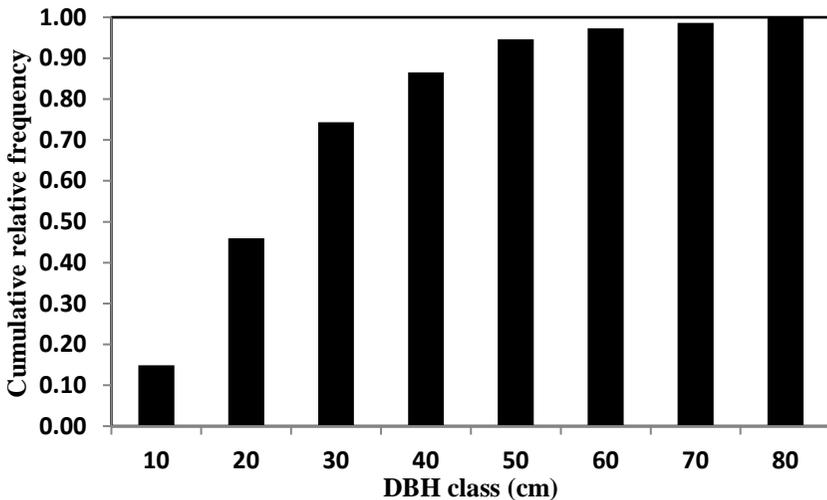
DBH class (cm)	Frequency	Relative frequency	Cumulative frequency
10- 11	11	0.15	11 ←
20- 11 + 23	23	0.31	34 ←
30- 34 + 21	21	0.28	55 ←
40	9	0.12	64
50	6	0.08	70
60	2	0.03	72
70	1	0.01	73
80	1	0.01	74
Total	74	1.00	



The value in the 20 cm DBH class would be described as: “In the area sampled, there were 34 trees with a DBH less than or equal to 25 cm.”

1d) Cumulative relative frequency histogram / table – Shows the relative frequencies of a variable class plus the relative frequencies of the classes below it (i.e., the percentage of observations that are less than or equal to the upper limit of each class). The Y-axis will range from 0 – 1. This is only valid for quantitative data.

DBH class (cm)	Frequency	Relative frequency	Cumulative frequency	Cumulative relative frequency
10 / 74	11	0.15	11	0.15
20 / 74	23	0.31	34	0.46
30 / 74	21	0.28	55	0.74
40	9	0.12	64	0.86
50	6	0.08	70	0.95
60	2	0.03	72	0.97
70	1	0.01	73	0.99
80	1	0.01	74	1.00
Total	74	1.00		

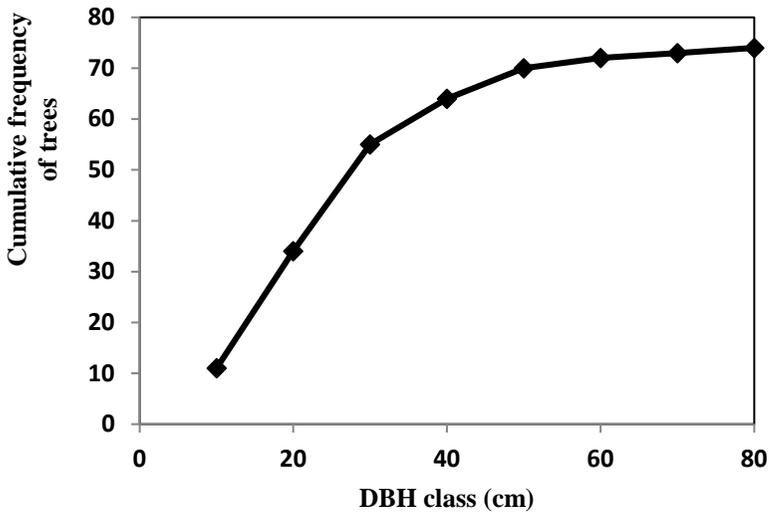


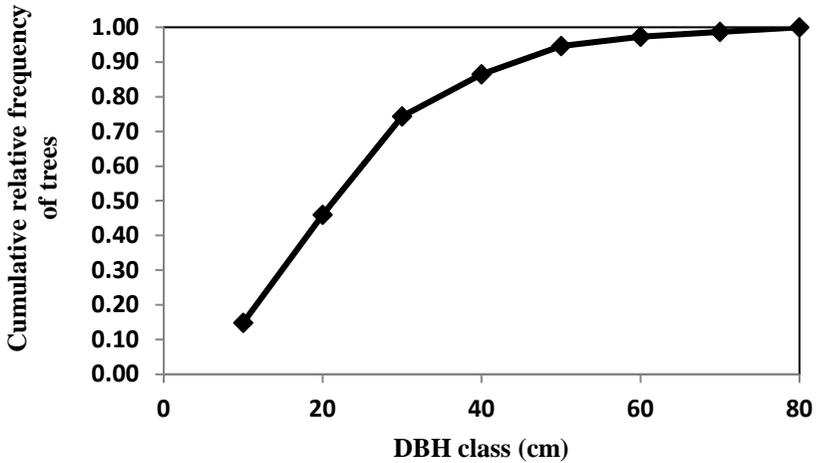
The value in the 20 cm DBH class would be described as:  
 “In the area sampled, 46% of the trees had a DBH less than or

equal to 25 cm.”

2) Ogives – These are the linear version of cumulative histograms (cumulative frequency and cumulative relative frequency).

DBH class (cm)	Cumulative frequency	Cumulative relative frequency
10	11	0.15
20	34	0.46
30	55	0.74
40	64	0.86
50	70	0.95
60	72	0.97
70	73	0.99
80	74	1.00

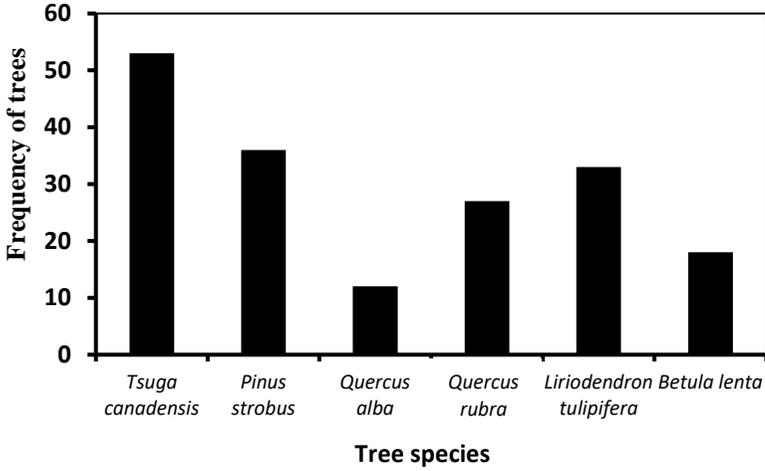




3) Bar graphs – These are similar to frequency histograms, but they are usually associated with categories common with qualitative data.

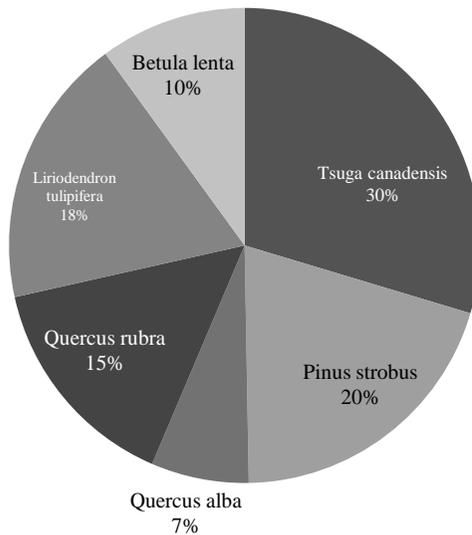
Tree species	Frequency
<i>Tsuga canadensis</i>	53
<i>Pinus strobus</i>	36
<i>Quercus alba</i>	12
<i>Quercus rubra</i>	27
<i>Liriodendron tulipifera</i>	33
<i>Betula lenta</i>	18

Associated with the data in the table above, an example bar graph is provided on the next page.



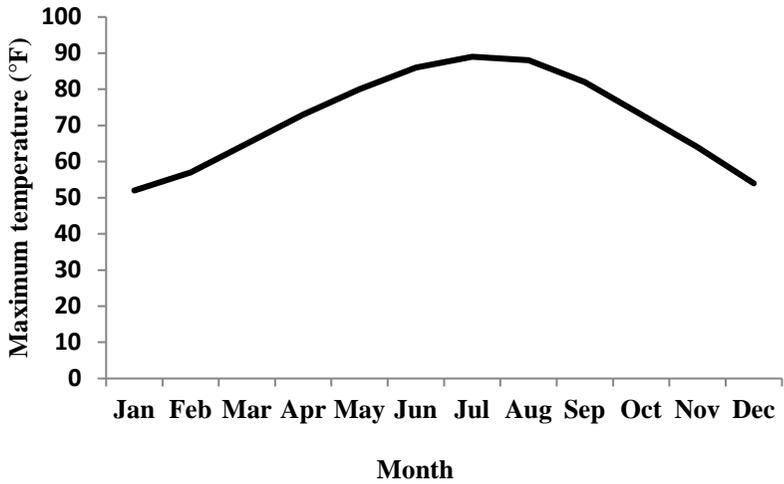
4) Pie charts – These are circular graphs showing how a total quantity is distributed among a group of categories (percentages / relative frequency).

### Proportion of tree species present

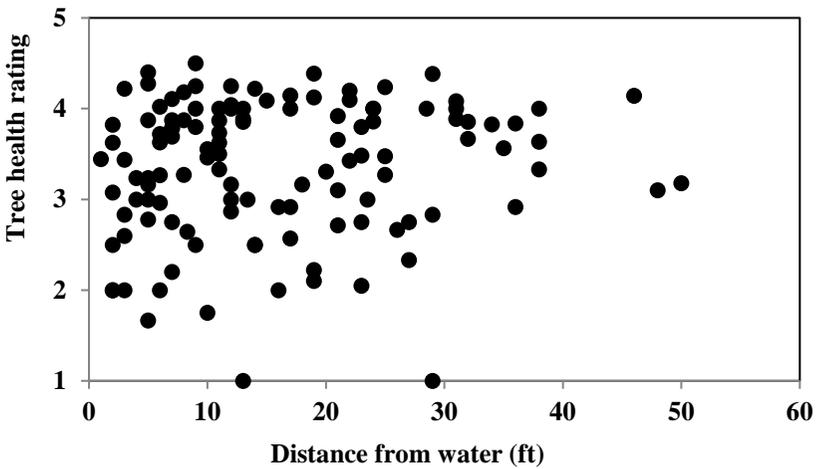


5) Line graph – These are used to display the change in one

variable in relation to another (often over time).



6) X/Y chart / Scatter plot – These are used to display the change in one variable in relation to another, as well as the variation in the relationship between X and Y.



### Defining classes for quantitative data

An important question to answer when describing quantitative data using a frequency or relative frequency distribution is whether you should subdivide the data. In one example above, 10-cm DBH classes were used. Clearly, you can choose any class width you would like, however some class widths may be more informative than others. The management or research question being considered, and the characteristics of the population of interest, usually dictates appropriate class limits. If a wildlife biologist is assessing the health of a bobcat population, then appropriate class determination could be to subdivide the sample into categories that are biologically relevant, such as year class.

- The first class interval should contain the smallest observation; the last class interval should contain the largest observation.
- The larger the number of observations, the larger the number of classes (common range is from 5 to 20 classes).
- Class intervals should be mutually exclusive (i.e., non-overlapping).
- Class intervals should be the same size.
- Class intervals should be exhaustive of the data (i.e., every observation should fall into some class).
- Be sensible; choose intervals which relate to the phenomenon under study.

For example, if you are considering deer age, and it is only meaningful to differentiate between deer <1 year, 1-2 years, 2-3 years, you shouldn't make classes with widths of 6 months, 2 years, etc.

To illustrate these concepts, consider the following data obtained from a sample of 80 largemouth bass taken from Lake Oconee.

Category	Frequency
Immature male	22
Immature female	28
Mature male	12
Mature female	18
Total	80

These data are qualitative. What is not as clear is whether the data are nominal or ordinal.

To summarize these data, we can use both the frequency and relative frequency concepts. In fact, the data presented in the table above are part of a frequency table. A complete frequency and relative frequency table for these data is:

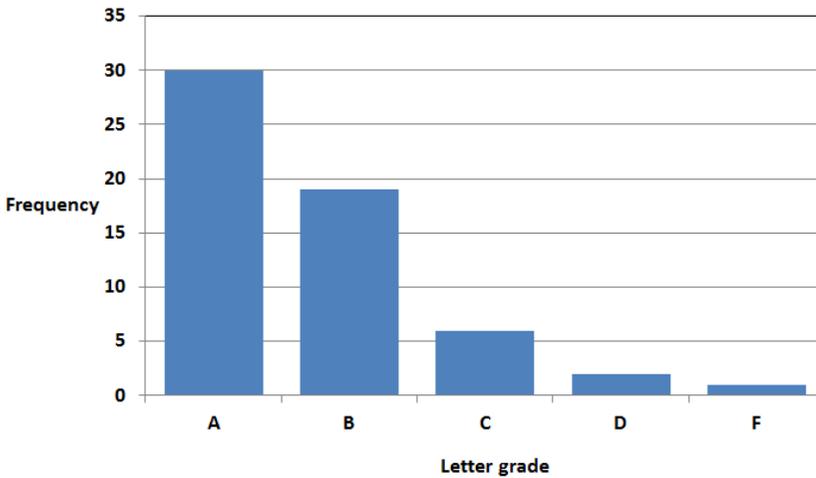
Category	Frequency	Relative frequency
Immature male	22	$22 / 80$
Immature female	28	$28 / 80$
Mature male	12	$12 / 80$
Mature female	18	$18 / 80$
Total	80	$80 / 80$

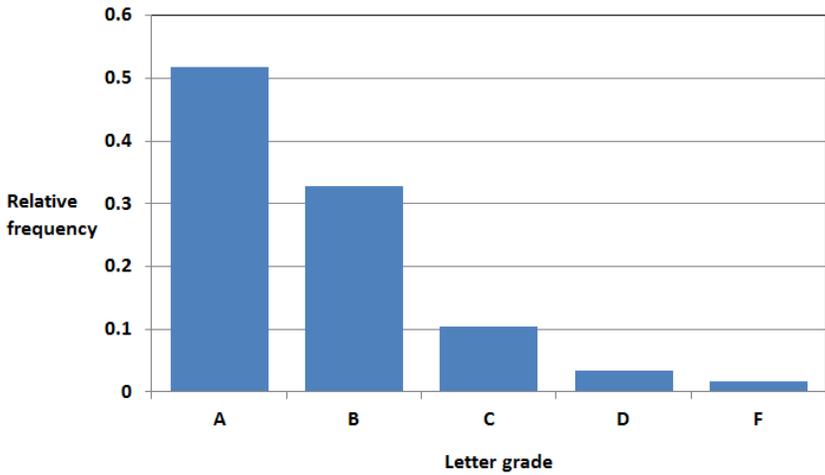
The frequency in each category can be summed across categories to obtain the total number of individuals sampled. Relative frequency is calculated by dividing each category's frequency by the total number of individuals. Note that relative frequency will sum to 1.0 when summed across all categories. Of course, we could have presented the decimal number rather than the fraction in the relative frequency column:

Category	Frequency	Relative frequency
Immature male	22	0.275
Immature female	28	0.350
Mature male	12	0.150
Mature female	18	0.225
Total	80	1.000

If we present frequency and relative frequency in a histogram, we have frequency and relative frequency histograms or bar charts. Two examples are shown below and on the next page.

*Frequency histogram*

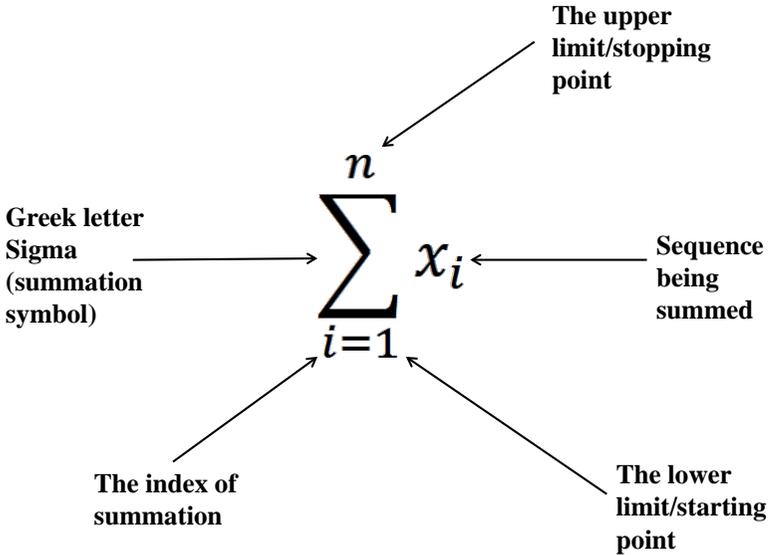


*Relative frequency histogram*

The shape of the frequency and relative frequency histograms are identical – the only difference is the scale of the y-axis.

### 10.3 SUMMATION NOTATION

When a study is conducted, often a large amount of sample data are generated. The data should be summarized before reporting. We typically do this by presenting our data as *sample statistics* (averages, ranges, modes, etc.). The sample statistics usually require the sums of many sets of numbers depicted by using *summation notation*.



The example above states that we sum (add) observation values  $x_1, x_2, \dots, x_n$  where each  $x$  is an individual observation of the variable of interest (weight of fish, height of trees, age of people surveyed, etc.), and where  $n$  equals the total number of observations.

**Example** We sampled 5 two-year old bears with individual weights of 145, 158, 133, 176, and 178 lbs, what is their sum?

$$\begin{aligned} \sum_{i=1}^n x_i &= \sum_{i=1}^5 x_1 + x_2 + x_3 + x_4 + x_5 \\ &= \sum 145 + 158 + 133 + 176 + 178 = 790 \text{ lbs} \end{aligned}$$

One of the biggest problems when using summation notation is incorrectly interpreting what data should be summed.

$x_i$
1
3
4
2
6

Given the data above, are these two notations the same?

(a)

$$\sum_{i=1}^n cx_i$$

(b)

$$c \sum_{i=1}^n x_i$$

Where  $c =$  a constant value of 0.25

(a)  $\sum_4 (0.25 * 1) + (0.25 * 3) + (0.25 * 4) + (0.25 * 2) + (0.25 * 6) =$   
 $\sum_4 0.25 + 0.75 + 1 + 0.5 + 1.5 =$

(b)  $0.25 * (\sum_4 (1 + 3 + 4 + 2 + 6)) =$   
 $0.25 * (16) =$

*Answer = Yes, they are the same*

Similarly, are these two notations the same?

(c)

$$\sum_{i=1}^n (x_i^2)$$

(d)

$$\sum_{i=1}^n (x_i)^2$$

$$(c) \quad \begin{aligned} \sum(1^2 + 3^2 + 4^2 + 2^2 + 6^2) &= \\ \sum(1 + 9 + 16 + 4 + 36) &= \\ 66 & \end{aligned}$$

$$(d) \quad \begin{aligned} \sum(1 + 3 + 4 + 2 + 6)^2 &= \\ 16^2 &= \\ 256 & \end{aligned}$$

*Answer = No, they are not the same*

A number of formulas used in this class will utilize summation notation so be sure to understand what you need to do. Remember, we deal with both qualitative and quantitative data in natural resource management. Graphical data description can be used for both types of data, however for quantitative data we can carry out numerical data description as well. In general, numerical data description techniques are more objective than graphical data description techniques.

### Algebraic rules for the summation operator

*Rule 1:* If  $c$  is a constant (i.e.,  $c$  does not have an index of summation) then:

$$\sum_{i=1}^n c = nc \text{ (sum } c \text{ } n \text{ times)}$$

For example:

$$\sum_{i=1}^3 5 = (5 + 5 + 5) = 3(5) = 15$$

$$\sum_{i=1}^4 5y = (5y + 5y + 5y + 5y) = 4(5y) = 20y$$

*Rule 2:* If  $c$  is a constant then:

$$\sum_{i=1}^n cy_i = c \sum_{i=1}^n y_i$$

For example,

$$\sum_{i=1}^{10} 8y_i = 8 \sum_{i=1}^{10} y_i$$

Often, multiple random variables are measured on each individual sample unit. For example, we may measure the diameter and height on a sample of tree stems. Thus, we have measured two random variables on each tree stem: diameter and height. For example, we may have measured wolf body weight (lbs) and body length (inches) from tip of nose to tip of tail. Thus, we can have two random variables measured on each sample wolf. We may refer to body weight as random variable  $y$  and body length as random variable  $x$ . Thus, for each animal we will have a  $x$  value and a  $y$  value. The data for both body length ( $x_i$ ) and body weight ( $y_i$ ) for 10 wolves are shown below. Therefore, the sum of body length values for these 10 animals is:

$$\sum_{i=1}^{10} x_i = 754 \text{ inches}$$

Animal	$x_i$ (Length in inches)	$y_i$ (Weight in lbs)
1	69	94
2	77	125
3	65	88
4	80	135
5	82	144
6	75	109
7	72	86
8	84	133
9	78	115
10	72	99
Total	754	1,128

*Rule 3:*  $\Sigma$  is a linear operator that works like all other linear operators in scalar algebra.

If we have 3 random variables ( $x, y, z$ ) measured on each of  $n$  individuals then we can write:

$$\sum_{i=1}^n (x_i + y_i + z_i) = \sum_{i=1}^n x_i + \sum_{i=1}^n y_i + \sum_{i=1}^n z_i$$

This simply shows that the sum of the linear quantity ( $x_i + y_i + z_i$ ) is simply the sum of the individual random variable sums. For the wolf data, we see that:

$$\sum_{i=1}^n (x_i + y_i) = \sum_{i=1}^n x_i + \sum_{i=1}^n y_i = 754 + 1128$$

Sometimes, we are interested in the sum of the “cross products” of two random variables. For example:

$$\sum_{i=1}^{10} x_i y_i = (x_1 y_1 + x_2 y_2 + \dots + x_{10} y_{10})$$

For the wolf data, we find that:

$$\sum_{i=1}^{10} x_i y_i = 86,076$$

## 10.4 MEASURES OF CENTRAL TENDENCY

Central tendency is one of the basic numerical data description measures is used to identify the *center* of the data. There are three basic quantities that are used:

- Sample mean
- Sample median
- Sample mode.

*Sample mean* – The arithmetic average of the values in a sample; the sum of all of the individual values divided by the number of observations.

If our random variable is  $y$  then we define the sample mean as:

$$\bar{y} = \left( \frac{\sum_{i=1}^n y_i}{n} \right)$$

Thus, to calculate the sample mean simply sum the values in the sample from 1 to  $n$  and divide by  $n$ . For the wolf data, we find that the average weight for our sample of 10 wolves is:

$$\bar{y} = \frac{1128}{10} = 112.8 \text{ lbs/animal}$$

Note that if we were to make an inference from this sample to the population of wolves from which the sample was drawn, we would infer that the population of wolves have an average body weight of 112.8 pounds.

*Sample median* – The exact center of the data when the data are sorted in order from the smallest numerical value to the largest

numerical value; 50% of the values are above the median, 50% are below. The median is useful when extreme values skew the sample mean. In general, the sample observation value to use for the median can be found in this manner:

$$\text{Observation to use} = \frac{n + 1}{2}$$

It is necessary to calculate the sample median slightly differently for sequences that have an even number of measurements and sequences that have an odd number of measurements.

*(a) Sample median for a sequence with an odd number of observations* - The value of the middle observation when the measurements are sorted from smallest to largest.

*(b) Sample median for a sequence with an even number of observations* - The average of the two middle observations when the measurements are sorted from smallest to largest.

For the wolf data, we have an even number of observations (10). Thus, to determine the sample median we first sort the data from small to large and then take the average of the middle two values:

$y_i$ (Weight in lbs)
86
88
94
99
109
115
125
133
135
144

Sample median =  $(109 + 115) / 2 = 112$  lbs.

*Sample mode* – The value of a sample set of measurements that occurs with the greatest frequency.

- Note 1 – if all sample values occur with the same frequency, the sample has no mode. For example, there is no mode for the wolf data.
- Note 2 – if two or more sample values have the same frequency, there are two or more modes.

**Example** We randomly sampled 10 bass from a local lake and measured their weights (lbs) which we then put in order from smallest to largest:

3 4 4 5 6 7 7 9 11 17

Mean: 
$$\bar{x} = \frac{\Sigma 3+4+4+5+6+7+7+9+11+17}{10}$$

$$\bar{x} = \frac{73}{10}$$

$$\bar{x} = 7.3 \text{ lbs}$$

Median: Observation to use =  $\frac{10+1}{2} = \frac{11}{2} = 5.5$

Because there is an even number of individuals in our samples, the median is the average of the 5<sup>th</sup> and 6<sup>th</sup> observation.

$$\text{Median} = (6 + 7) / 2 = 6.5 \text{ lbs}$$

Mode: The values 4 and 7 are repeated twice in the sample, making both of them the mode.

## 10.5 MEASURES OF RELATIVE STANDING

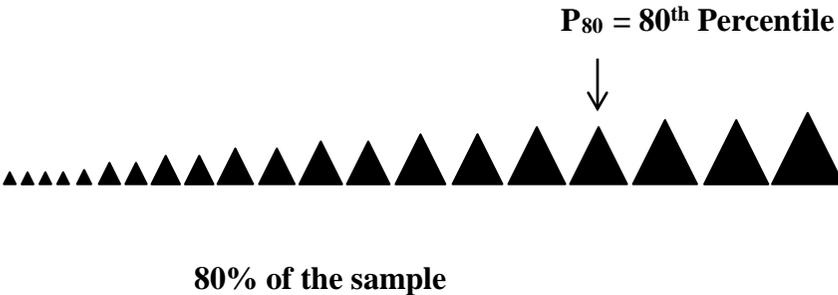
This type of measure indicates the relative position in a sequence of data values when the values are sorted from low to high.

Various quantities are used for this measure:

- Percentiles
- Median (described earlier)
- Quartiles
- Deciles.

### *Percentiles*

The  $p^{\text{th}}$  percentile of a sequence of data is the value such that at least  $p$  percent of the data points take on this value or less and  $(100-p)$  percent of the data points take on this value or greater.



There are many algorithms used to determine percentiles for sample data. We will use the following approach:

- Sort the data sequence from smallest to largest
- Compute the position ( $i$ ) in the sorted sequence for the percentile of interest as:  

$$i = (p / 100) n$$
 where:  
 $p$  = percentile of interest  
 $n$  = number of data points in the sequence
- If  $i$  is not an integer, then the next highest integer value

denotes the position in the sorted sequence of the  $p^{\text{th}}$  percentile

- If  $i$  is an integer, then the  $p^{\text{th}}$  percentile is found by taking the average of the data values found in positions  $i$  and  $i + 1$  of the sorted sequence.

Therefore, in general, the sample observation value to use as the percentile value is determined with:

$$\text{Observation to use} = \left( \frac{p}{100} \right) n$$

**Example** The 20<sup>th</sup> percentile ( $P_{20}$ ) of the wolf data is determined as:

$$\text{Observation to use} = (20 / 100) 10 = 2$$

Thus, the position of  $P_{20}$  is 2 (an integer value).  $P_{20} = (88+94) / 2 = 91$  lbs. (e.g. take the data values in positions 2 and 3 and average them). So, we would infer that 20 percent of the animals in the population that was sampled would have body weight of 91 pounds or less.

**Example** The 63<sup>rd</sup> percentile ( $P_{63}$ ) of the wolf data is determined as:

$$\text{Observation to use} = (63 / 100) 10 = 6.3$$

Thus, the position of  $P_{63}$  is 6.3 (non-integer).  $P_{63}$  is then found in position 7 (the next highest integer value) in the sorted sequence which is 125 lbs. So, we infer that 63 percent of the animals in the population that was sampled have a body weight of 125 pounds or less.

You should note that  $P_{50}$  is also the median.

### *Deciles and Quartiles*

Deciles and quartiles are simply specific percentiles that are given different names.

Deciles divide the data into 10 equal groups and are denoted as:  $D_1, D_2, D_3, \dots, D_9$ . These values correspond to  $P_{10}, P_{20}, P_{30}, \dots, P_{90}$ .

Quartiles divide the data into 4 equal groups and are denoted as  $Q_1, Q_2$  and  $Q_3$ . These values correspond to  $P_{25}, P_{50}$  and  $P_{75}$ .

One can calculate deciles and quartiles in the same manner as one calculates the corresponding percentiles.

For quartiles, the following steps might be used:

Step 1. Determine the median which will divide the data into 2 equal sets. This is  $Q_2$  or the 50<sup>th</sup> Percentile.

Step 2. Determine the median for each half. These are the lower ( $Q_1$ ) and upper ( $Q_3$ ) Quartiles (or the 25<sup>th</sup> and 75<sup>th</sup> Percentiles).

## 10.6 MEASURES OF DISPERSION

In addition to measures of central tendency and relative standing, we often need to quantify the spread or variability of the data. In general, measures that represent variability of data are classified as measures of dispersion. Five such measures are:

- Range
- Inter-quartile range
- Sample variance
- Sample standard deviation
- Coefficient of variation (CV)

### *Range*

The range is a very basic and simple measure of the amount of dispersion in data. It is simply defined as the difference between the largest and smallest values in the sample data of interest. For the wolf weight data the range is:

$$\text{Range} = 144 - 86 = 58 \text{ lbs}$$

### *Inter-quartile range (IQR)*

The inter-quartile range is defined as the difference between the third quartile ( $Q_3$ ) and the first quartile ( $Q_1$ ) and hence represents the spread of the middle 50% of the data. Recall that  $Q_3$  is simply the 75<sup>th</sup> percentile ( $P_{75}$ ) and  $Q_1$  is the 25<sup>th</sup> percentile ( $P_{25}$ ). To determine the IQR for the wolf weight data, we first must calculate  $Q_1$  and  $Q_3$ :

To calculate  $Q_1$ , realize that it is the same as  $P_{25}$  and simply determine this value. To do this, we first calculate the position in the sorted data that  $P_{25}$  corresponds to:

$$i = (25 / 100) 10 = 2.5$$

Thus, the position in the sorted sequence of wolf weights is 2.5 which is a non-integer. Therefore,  $Q_1$  ( $P_{25}$ ) is found in the 3<sup>rd</sup> position of the sorted sequence:

$$Q_1 = P_{25} = 94 \text{ lbs}$$

To calculate  $Q_3$ , we follow the same procedure once we realize that it is equivalent to  $P_{75}$ . The position of  $P_{75}$  for our wolf weight data is  $i = 7.5$ . Thus, we find  $Q_3$  ( $P_{75}$ ) in position 8 of the sorted sequence which is:

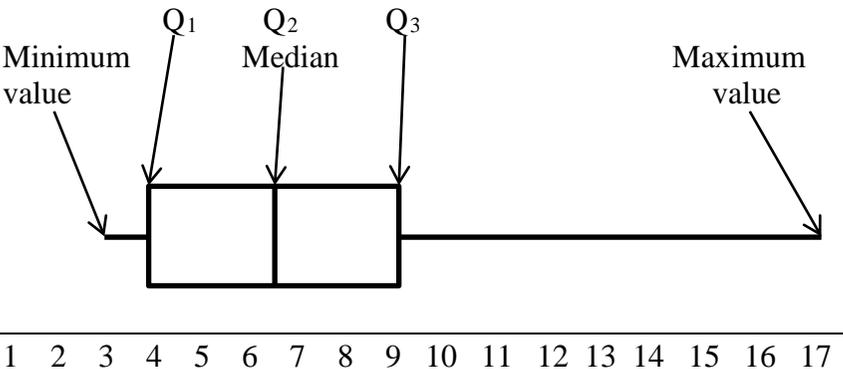
$$Q_3 = P_{75} = 133 \text{ lbs}$$

Thus, we can now easily determine the IQR to be:

$$\text{IQR} = (Q_3 - Q_1) = (133 - 94) = 39 \text{ lbs}$$

Consequently, we now know that the middle 50% of the data has a spread of 39 lbs.

Range and Inter-Quartile Range are used to create *Box Plots*, where the box represents the IQR and the “whiskers” typically encompass the entire range (unless there are extreme outliers, which may be included as dots beyond the whiskers).



Box plots also help to determine skewness of the data. For

instance, if  $Q_2$  is closer to  $Q_1$  then the data is skewed to the right. If  $Q_2$  is closer to  $Q_3$  then the data is skewed left (i.e., the position of median within the box determines the skewness). We can also see the variability of the data from the length of the box.

### *Variance*

Sample variance and standard deviation are the most often used measures of dispersion for sample data. Both quantities are used to represent data variability. It turns out that the sample variance has a great deal of utility for many types of statistical inference procedures and hence it is very important to understand what it is and how it is calculated.

To understand sample variance, it is first necessary to realize that variability exists in nature. That is, all biological organisms exhibit variability in their characteristics. For example, if we are interested in the weight of black bears we may choose to capture and weigh 50 animals. Each animal will have a different weight. Thus, we say that the weight of black bears is a “random” variable. It is random because each animal differs in its weight. Almost all measures of size associated with biological organisms are random variables and hence these measures have variability.

Sample variance is simply an attempt to quantify the amount of variability in any given random variable. Assume that we have a random variable denoted as  $y_i$ . Further assume that we have measured  $n$  individuals for this random variable (e.g., body weight, stem height, etc.) and we have the following sequence established for this random variable:

$$y_1, y_2, y_3, \dots, y_n$$

Sample variance is denoted as  $s_y^2$  and is determined using one of the two following equations:

$$s_y^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1}$$

$$s_y^2 = \frac{\sum y_i^2 - \left( \frac{(\sum y_i)^2}{n} \right)}{n-1}$$

A close look at this definition of sample variance reveals that the numerator tells us to determine the distance that each individual  $y_i$  value is from the mean of the values, square this distance and add all such squared distances together. Finally, we divide the sum of the squared distances by  $(n-1)$ . Thus, sample variance is essentially equivalent to the average of the squared distances between the individual  $y_i$  values and their mean. Because these distances are squared, the units of variance are squares of the original units. So if we have measured the weight of an animal in pounds, its mean is in units of pounds but its sample variance is in units of square pounds.

To illustrate variance calculation we will make use of the wolf weight data. To use the calculation equation for sample variance, we must know the following quantities for our sample data:  $n$ ,

$\sum_{i=1}^n y_i$ , and  $\sum_{i=1}^n y_i^2$ . For the wolf weight data we have:

$$n = 10$$

$$\sum_{i=1}^n y_i = 1,128 \text{ lbs}$$

$$\sum_{i=1}^n y_i^2 = 131,158 \text{ lbs}^2$$

Now, we take these values and insert them into the sample variance calculation equation:

$$s_y^2 = \frac{131,158 - \left( \frac{(1128)^2}{10} \right)}{9}$$

$$s_y^2 = 435.5 \text{ lbs}^2$$

- The units of sample variance are squares of the original units used for the random variable of interest.
- If we were to increase our sample size ( $n$ ), it may decrease your variance.

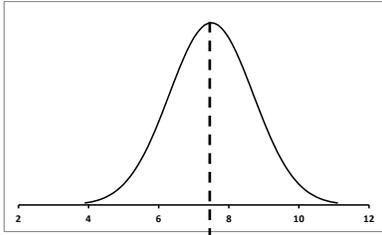
### *Standard deviation*

Often times we do not want to deal with squares of the original units, therefore we take the square root of sample variance to return the units to the original units. When we take the square root of the variance, we obtain the sample standard deviation which is denoted as  $s_y$ . Thus, for the wolf weight data we can easily find the sample standard deviation to be:

$$s_y = \sqrt{s_y^2} = \sqrt{435.5} = 20.9 \text{ lbs}$$

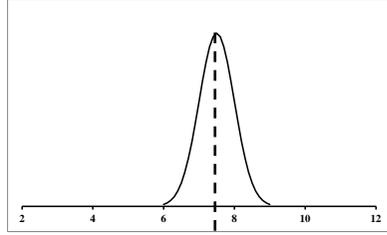
The standard deviation helps explain the spread of the data we collected around the mean. This spread can be visualized in normal distribution curves.

$\bar{x} = 7.5$  lbs., SD = 1.2 lbs.



**7.5**

$\bar{x} = 7.5$  lbs., SD = 0.5 lbs.



**7.5**

As you can see, the curve on the right, with the lower SD, is more narrow, indicating that the data are less spread out and closer to the mean than the curve on the left.

### *Coefficient of variation (CV)*

The last measure we use to represent the variability within sample data is the coefficient of variation (CV). This value is easily calculated once the sample mean and sample standard deviation are calculated:

$$CV = \frac{s_y}{\bar{y}}(100)$$

Thus, CV is simply a measure of how large the sample standard deviation is relative to the sample mean that is presented as a percentage. Clearly, the ratio of the sample standard deviation over the sample mean is a unitless proportion since both measures have the same units. When this unitless proportion is multiplied by 100, it is converted to a percentage. Therefore, the CV tells us how large the sample standard deviation is relative to the sample mean on a percentage basis.

CV is useful for comparing the variability found within different sets of sample data. It should be noted that sample variance and

sample standard deviation represent the amount of variability within sample data, however these quantities are influenced by the scale of measurement and cannot readily be used to compare variability from one set of sample data to another.

**Example** The average height was calculated for 2 loblolly pine stands of different ages. We want to know which stand has more height variation.

Stand 1

$$\bar{y} = 40 \text{ ft}$$

$$s_y = 4.4 \text{ ft}$$

Stand 2

$$\bar{y} = 5.9 \text{ ft}$$

$$s_y = 0.649 \text{ ft}$$



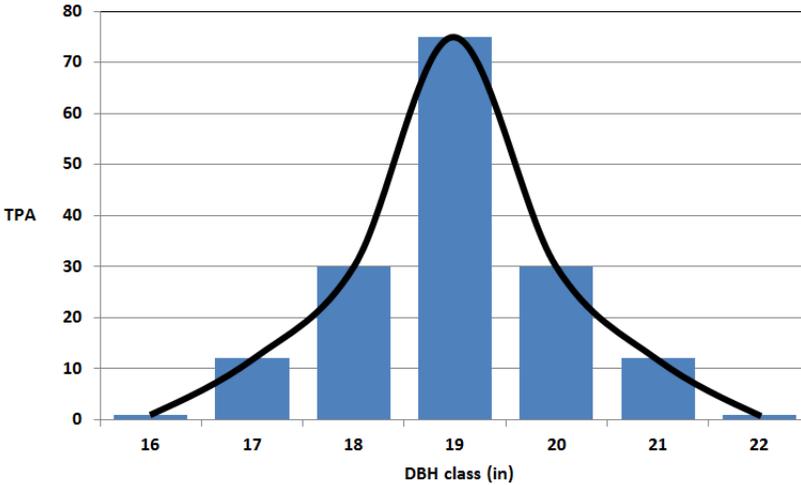
$$\text{Stand 1 CV} = (4.4 / 40) 100 = 11\%$$

$$\text{Stand 2 CV} = (0.649 / 5.9) 100 = 11\%$$

The stands are equally variable (The SD is 11% of the mean).

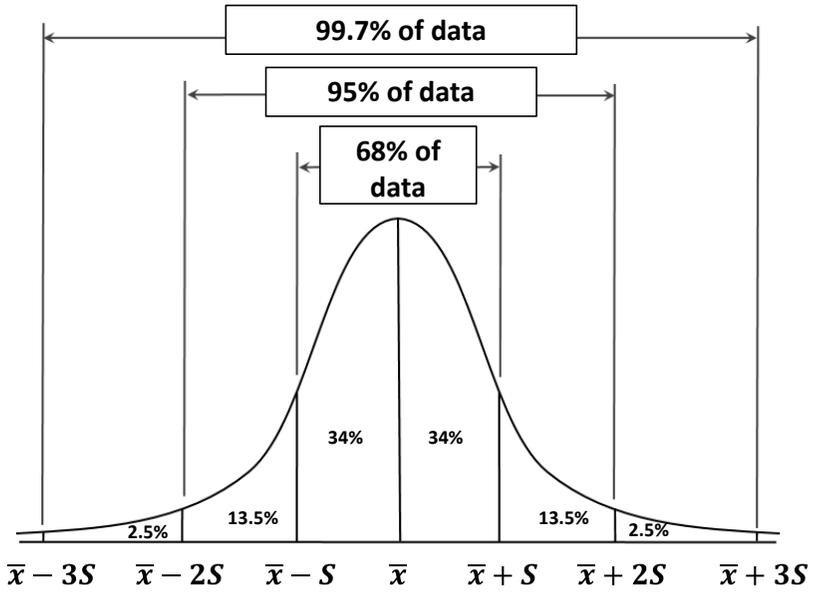
### *Empirical Rule*

In nature, it has been observed that many random variables come from populations that have a bell-shaped population distribution. In such a population, most of the individual values of the random variable fall near the sample mean with fewer values falling further away from the sample mean.



The empirical rule allows us to make statements about what proportion of sample data fall within a certain number of standard deviations of the sample mean if the data form a bell-shaped distribution, as shown above. Specifically, the rule states:

- Approximately 68% of the data should be within  $\pm 1 s_y$  (standard deviation) of the sample mean
- Approximately 95% of the data should be within  $\pm 2 s_y$  of the sample mean
- Almost all (99.7%) of the data should be within  $\pm 3 s_y$  of the sample mean

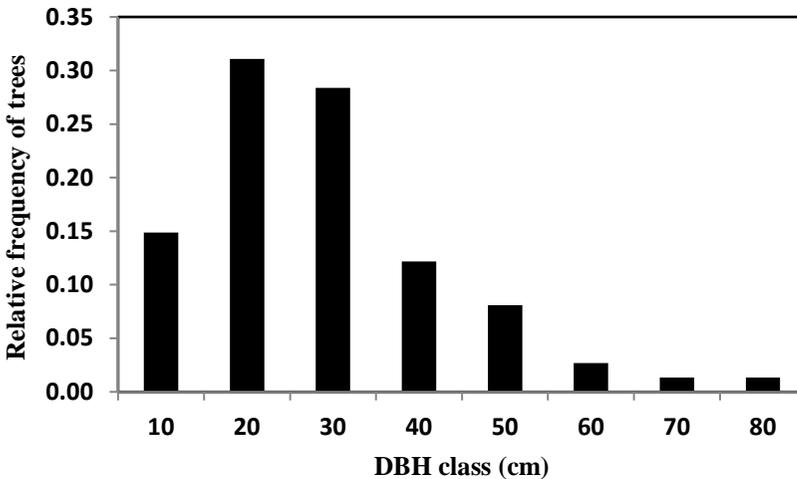


## 10.7 FREQUENCY / PROBABILITY DISTRIBUTIONS

We will now shift our discussion from *descriptive statistics* (measures of central tendency, relative standing, and dispersion) to *inferential statistics* (expanding conclusions from the sample to the population of interest). We must first consider some concepts associated with probability.

*Probability* ( $P[A]$ ) is the likelihood that a particular event or value ( $A$ ) will occur. We typically determine a probability based on the relative frequency distribution of our sample. The probability of an event will always be between 0 and 1, and the sum of all probabilities will be equal to 1.

**Example** What is the probability that a randomly selected sample tree will be in the 30 cm DBH class (DBH between 25 and 35 cm)?

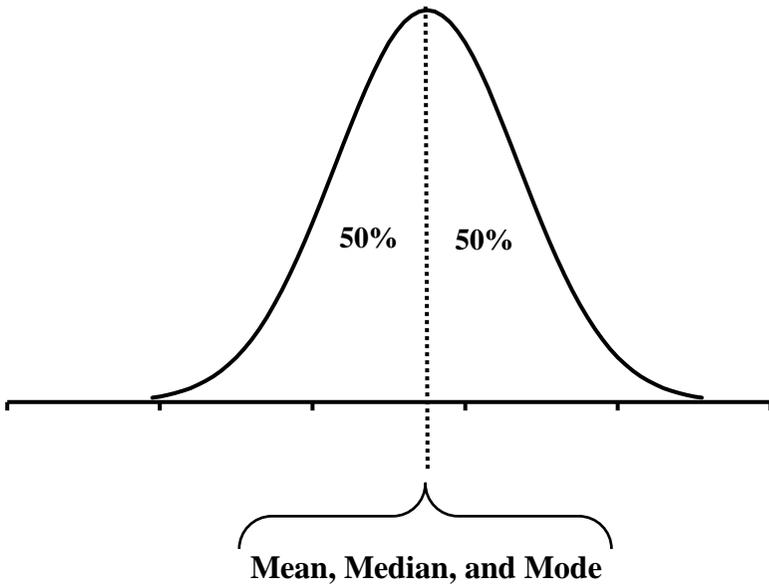


$$P[25 \leq x_i \leq 35] = 0.28$$

The probability is 28% that a random sample tree would be in the 30 cm DBH class.

*Normal probability distribution* – This is a continuous probability distribution where:

1. The shape is a symmetrical / bell-shaped distribution.
2. The measures of central tendency (mean, median, and mode) are about the same.
3. The total area under the curve is equal to 1.0.
4. The variable of interest has an infinite range (the ends of the distribution never really touch the horizontal axis).



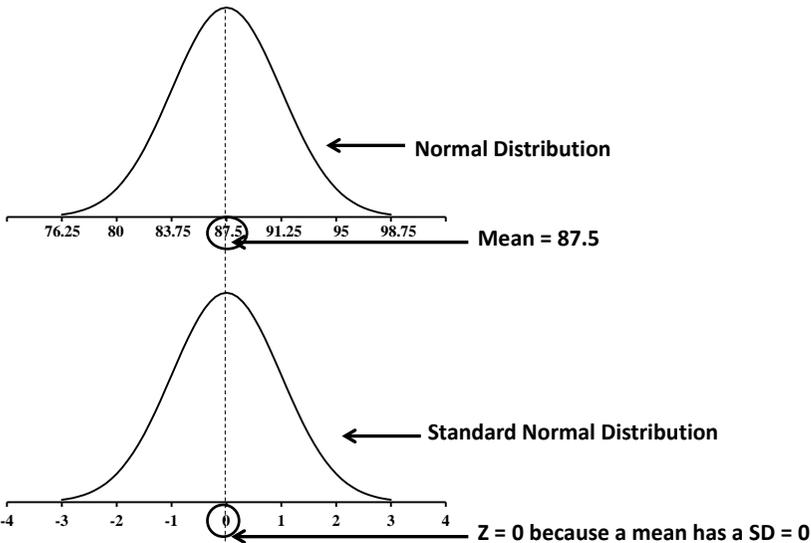
- The mean and the standard deviation of a data set affect the normal distribution, with the mean determining the center / peak and the standard deviation determining the height and width of the curve.
- If the standard deviation is large, the curve will be short and wide.
- If the standard deviation is small, the curve will be tall and narrow.
- Many statistical methods require the assumption of a normal distribution.

*Standard normal Z-score* – This value represents the number of standard deviations an observation is above or below the mean ( $\mu$ ). A Z-score of 0 represents a value equal to the mean, a Z-score of 1 represents a value 1 SD *above* the mean, a Z-score of -1 represents the value 1 SD *below* the mean, etc.

$$Z = \frac{x - \mu}{\sigma}$$

- A Z-score should only be used when the population mean and SD are known, but often this is unrealistic. The sample statistics can be substituted (usually if  $n > 30$ ).

A Z-score can be correlated to a probability using a *Standard normal distribution / Z distribution curve* (See Appendix E). This table presents values concerning a standardized normal distribution, where the X-axis is the number of standard deviations away from the mean (Z-scores), the mean is equal to zero (because the mean is zero standard deviations away from the mean), and the standard deviation is 1.



Tips for interpreting a Z-table:

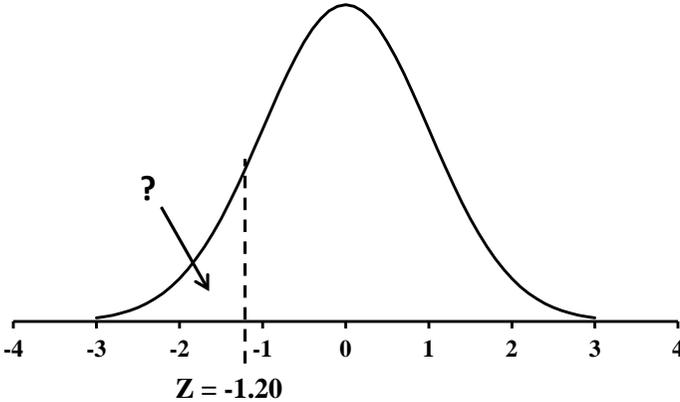
1. If you want to know the proportion of the data that are  $\pm$  a particular SD (Z-score) from the mean, you will need to double the value given in the table (because the table only looks at half of the curve).
2. If the Z-score is positive and you want to know the proportion of the data that are above the Z-score of interest, subtract the value given from 0.50 (because half of the curve is 50% of the data).
3. If the Z-score is negative and you want to know the proportion of the data that are above the Z-score, add 0.50 to the value given.

The left column of the table corresponds to Z-score up to the tenths decimal place (##) and the top row of the table corresponds to the number in the hundredths place of the Z-score (0.0#).

Z	.00	.01	.02	.03	.04	.05
0.0	.0000	.0040	.0080	.0120	.0160	.0199
0.1	.0398	.0438	.0478	.0517	.0557	.0596
0.2	.0793	.0832	.0871	.0910	.0948	.0987
0.3	.1179	.1217	.1255	.1293	.1331	.1368
0.4	.1554	.1591	.1628	.1664	.1700	.1736
0.5	.1915	.1950	.1985	.2019	.2054	.2088
0.6	.2257	.2291	.2324	.2357	.2389	.2422
0.7	.2580	.2612	.2642	.2673	.2704	.2734
0.8	.2881	.2910	.2939	.2967	.2995	.3023
0.9	.3159	.3186	.3212	.3238	.3264	.3289
1.0	.3413	.3438	.3461	.3485	.3508	.3531
1.1	.3643	.3665	.3686	.3708	.3729	.3749
1.2	.3849	.3869	.3888	.3907	.3925	.3944
1.3	.4032	.4049	.4066	.4082	.4099	.4115
1.4	.4192	.4207	.4222	.4236	.4251	.4265
1.5	.4332	.4345	.4357	.4370	.4382	.4394
1.6	.4452	.4463	.4474	.4484	.4495	.4505
1.7	.4554	.4564	.4573	.4582	.4591	.4599
1.8	.4641	.4649	.4656	.4664	.4671	.4678
1.9	.4713	.4719	.4726	.4732	.4738	.4744
2.0	.4772	.4778	.4783	.4788	.4793	.4798
2.1	.4821	.4826	.4830	.4834	.4838	.4842
2.2	.4861	.4864	.4868	.4871	.4875	.4878

**Example** If  $Z\text{-score} = 2.15$ , 48.42% of the data are between 0 and 2.15 SD around the sample mean.

**Example** Using mid-term exam score data, we found that John Smith received a score of 83. This corresponded to a  $Z$ -value of  $-1.20$ . What proportion of the class scored below an 83?



1) Determine the proportion of the curve between 0 and 1.20.

$$P[0 \leq Z \leq 1.2] = 0.3849$$

The probability that a standard  $Z$ -score is between 0 and 1.2 is 38.49%. Thus 38.49% of the class scored between 83 and 87.5 on the exam.

2) Determine the proportion of the curve that is less than  $-1.20$ .

Since half of the curve will always equal 50%, we need to subtract 0.3849 from 0.50 to determine the portion of the curve less than  $-1.20$ .

$$P[Z \leq -1.20] = 0.1151$$

11.51% of the class received a score less than 83.

**Example** We found an article that gives the average number of acres burned per year in Georgia. In the paper, we found the following information:

- Mean ( $\mu$ ) = 4,300 acres per year
- Standard deviation ( $\sigma$ ) = 750 acres per year
- Data was normally distributed.

What is the probability that in any given year, more than 5,000 acres will be burned?

First, calculate the Z-score associated with 5,000 acres per year.

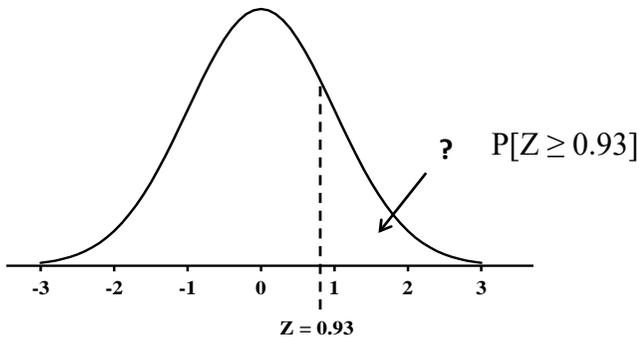
$$Z = \frac{x - \mu}{\sigma}$$

$$Z = (5,000 - 4,300) / 750$$

$$Z = 700/750$$

$$Z = 0.93$$

Next, determine the probability is that Z is greater than this value:

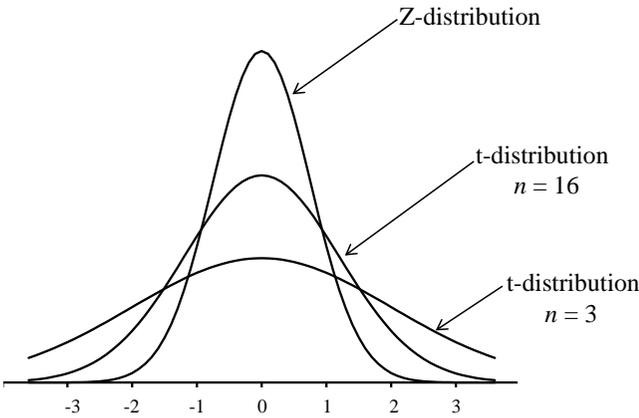


This value came from the Z-table (by lining up 0.9 and 0.03), and tells us the area under the curve from 0 to 0.93 is 32.38%. Since we need to know the area *greater* than 0.93, we need to subtract this value from 50%.

$$P[Z \geq 0.93] = 0.50 - 0.3238 = 0.1762$$

Answer: There is a 17.62% chance that an average of over 5,000 acres will be burned in any given year.

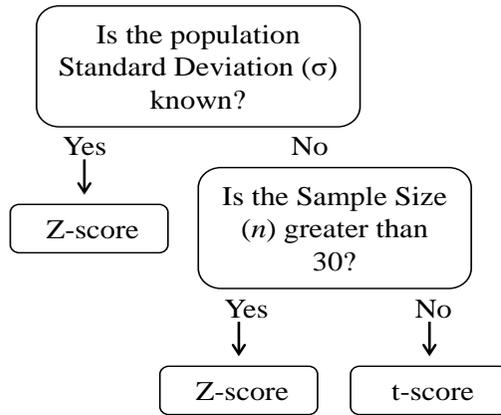
(Student's) *t*-distribution – This is a probability distribution that is used when the sample size is too small (usually  $n < 30$ ) or when the population standard deviation ( $\sigma$ ) is unknown. Think of it as the cousin to the *Z*-distribution – the *t*-distribution is bell shaped and centered at zero, but the standard deviation is not a constant of 1. Instead, the standard deviation varies based on the sample size, but will be larger than 1 until the sample size is larger ( $n > 30$ ); the *t*-distribution will be flatter and more spread out (higher variance). As the sample size increases, the *t*-distribution will more closely approximate the *Z*-distribution.



Similar to the *Z*-distribution, a table exists to help us understand the proportion of area under the *t*-distribution curve associated with a *t*-score (see Appendix E).

Degrees of Freedom	Upper Tail Areas					
	.25	.10	.05	.025	.01	.005
1	1.0000	3.0777	6.3138	12.7062	31.8207	63.6574
2	0.8165	1.8856	2.9200	4.3027	6.9646	9.9248
3	0.7649	1.6377	2.3534	3.1824	4.5407	5.8409
4	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
5	0.7267	1.4759	2.0150	2.5706	3.3649	4.0322
6	0.7176	1.4398	1.9432	2.4469	3.1427	3.7074
7	0.7111	1.4149	1.8946	2.3646	2.9980	3.4995
8	0.7064	1.3968	1.8595	2.3060	2.8965	3.3554

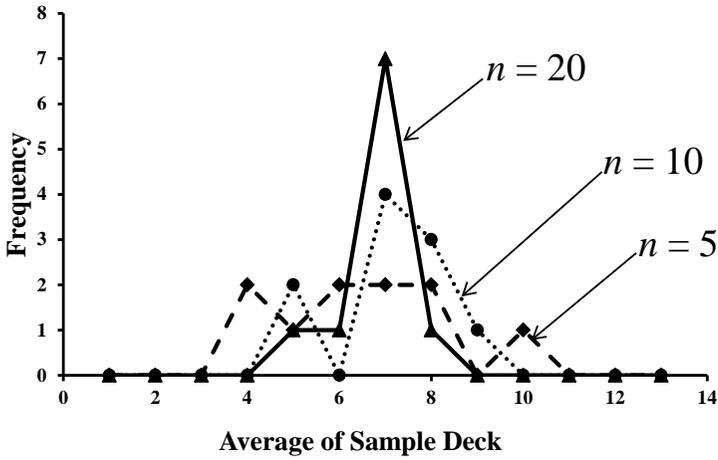
You should ensure that you are using the appropriate distribution / score based on what you know or collected:



*Central Limit Theorem* – When we draw multiple samples of the same size from the population and calculate a mean for each sample, the distribution of the sample means will follow a normal distribution (regardless of the distribution of the population). Also, as the number of samples gets larger, the sample means will cluster around the true population mean ( $\mu$ ).

**Example** Assume that our population is 20 decks of cards ( $N = 1040$  cards) with each card having a value between 1 and 13. The frequency distribution of the population would NOT be normally distributed because there are the same number of cards of each value, and therefore the same probability of selection.

However, if we randomly selected a sample of cards from 10 of the decks ( $n = 5, 10, \text{ or } 20$  cards), and calculated the average for each deck, we would see that these averages begin to follow a normal distribution.



$$\mu = 7.0$$

$$\bar{x} = 6.8$$

Also, even though the sample means for each deck were not the same, the means clustered around the true population mean ( $\mu = 7$ ) as we increased our sample size.

## 10.8 MEASURES OF PRECISION AND ACCURACY

Often, we want to present our estimates (e.g., the sample mean) with values that describe the reliability of our estimates. Two of these are important:

- **Standard error** – A measurement of the accuracy of the sample estimate as it represents the population. It indicates how close the sample data are to the estimated population parameter.
- **Confidence interval** – The range around a sample estimate that describes the precision of the estimate. It indicates how confident we are that our sample estimate represents the population of interest.

*1. Standard error* – The standard error is the standard deviation of the sample means (based on the population mean), or the variability of the mean from one sample to the next.

$$s_{\bar{y}} = \left( \frac{s_y}{\sqrt{n}} \right)$$

- The standard error decreases as the sample size increases.

*2. Confidence intervals* – These provide an estimate of the interval (with an associated confidence level) in which the population parameter may fall.

$$CI = \bar{x} \pm Z_{(1-\alpha)/2} (s_{\bar{y}}) \quad \text{or} \quad CI = \bar{x} \pm t_{n-1,(\alpha/2)} (s_{\bar{y}})$$

**Example** Calculate the 90% confidence interval for a sample ( $n = 26$ ) of fish weights with a mean ( $\bar{x}$ ) of 8.7 lbs and a sample standard error ( $s_{\bar{y}}$ ) of 1.3 lbs.

- 1) Determine which distribution table is necessary. In this example, we don't know the population SD and our sample size is  $< 30$ , so we will use the formula based on the t-distribution:

$$CI = \bar{x} \pm t_{n-1,(\alpha/2)} \left( s_{\bar{y}} \right)$$

- 2) Determine the degrees of freedom.  $26 - 1 = 25$
- 3) Determine  $\alpha$ . Since we are calculating a 90% CI, we are willing to accept 10% error, so  $\alpha = 0.10$  ( $\alpha/2 = 0.05$ ).
- 4) Find the t-value associated with the  $\alpha$  and  $df$

$$t_{25,0.05} = 1.7081$$

The 90% CI =  $8.7 \pm 1.7081 (1.3)$

Therefore the 90% CI =  $8.7 \pm 2.22$  lbs, or  $6.48 \leq \mu \leq 10.92$ .

In sum, we are 90% confident that the true population average weight of fish is between 6.48 and 10.92 lbs.

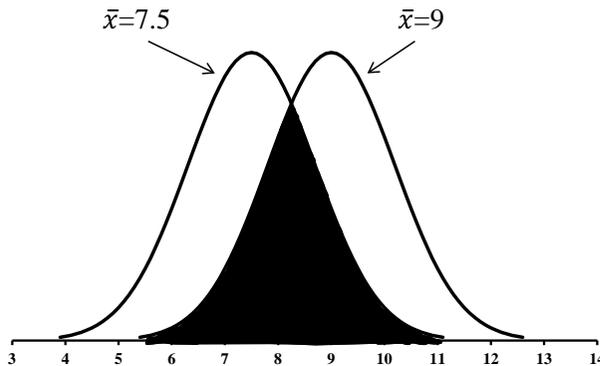
## 10.9 COMPARISON OF MEANS

Oftentimes, we are interested in comparing means from two different populations to determine if they are significantly different. For example,

- Is one loblolly pine stand worth significantly more than another?
- Are the sizes of fish from a healthy stream significantly different than those from an unhealthy stream?
- Are turkey populations significantly lower at sites that have had prescribed burns?

Two means can be numerically close together but significantly different because the SD is small for each. Conversely, two means which are numerically far apart may not be significantly different because the SD is large for each.

When comparing means, the overlap of the data plays a role in determining whether or not a significant difference exists:



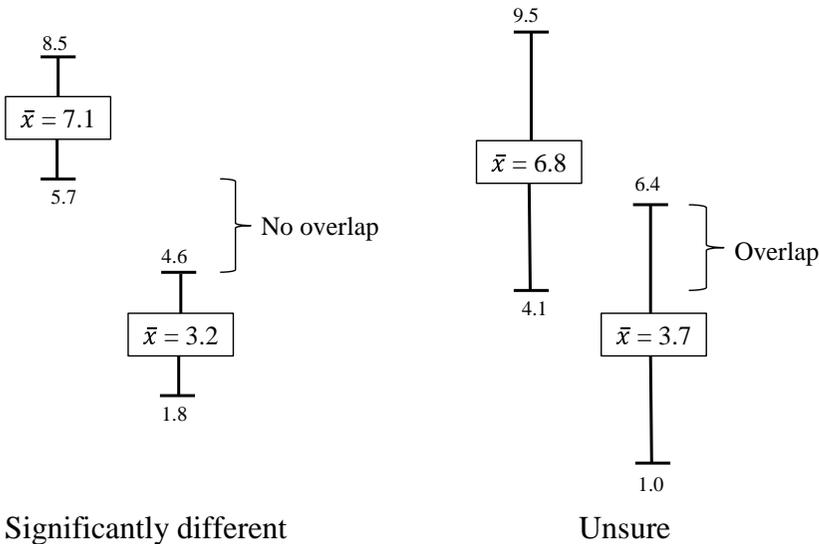
The overlap helps determine the probability that the two data sets are from the same distribution (i.e., that they are not significantly different). This probability increases as the overlap increases.

A statistically significant difference between estimates implies that the difference between them is unlikely to have occurred by chance.

### Comparing sample means using confidence intervals

You can use confidence intervals as a quick way to check for statistical significance.

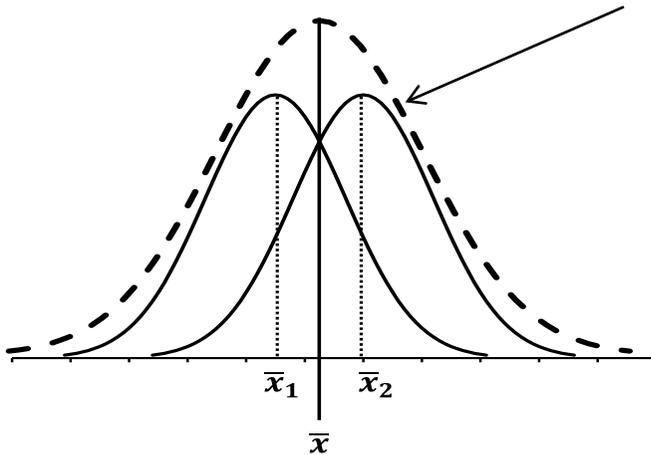
- If there is no overlap between the intervals, the means are significantly different.



### Student's $t$ -test

Many studies involve comparing the same parameter between two different populations (e.g., is there a difference in average incidence of deer collisions between Georgia and Michigan? Or is there a difference in average exam grades between Monday and Tuesday's labs?)

When comparing 2 means, we use a sampling distribution for the *difference in means*. Basically, we want to compare 2 different samples to determine if they come from the same population:

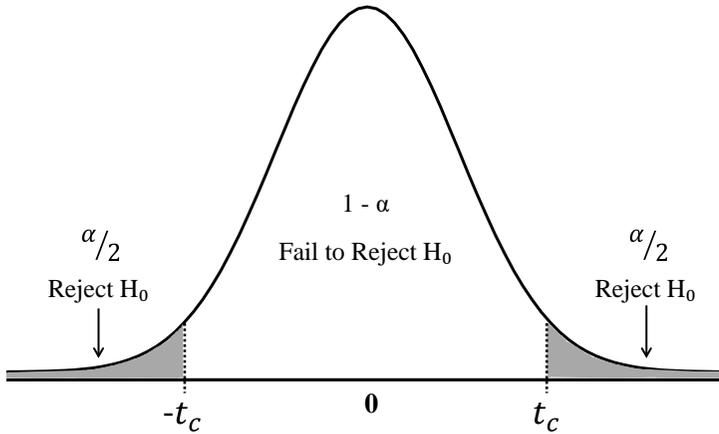


The mean of this distribution ( $\bar{x}$ ) is the difference between the two sample means ( $\bar{x}_1 - \bar{x}_2$ ).

A 2-sample  $t$ -test is appropriate only if the samples meet the following criteria:

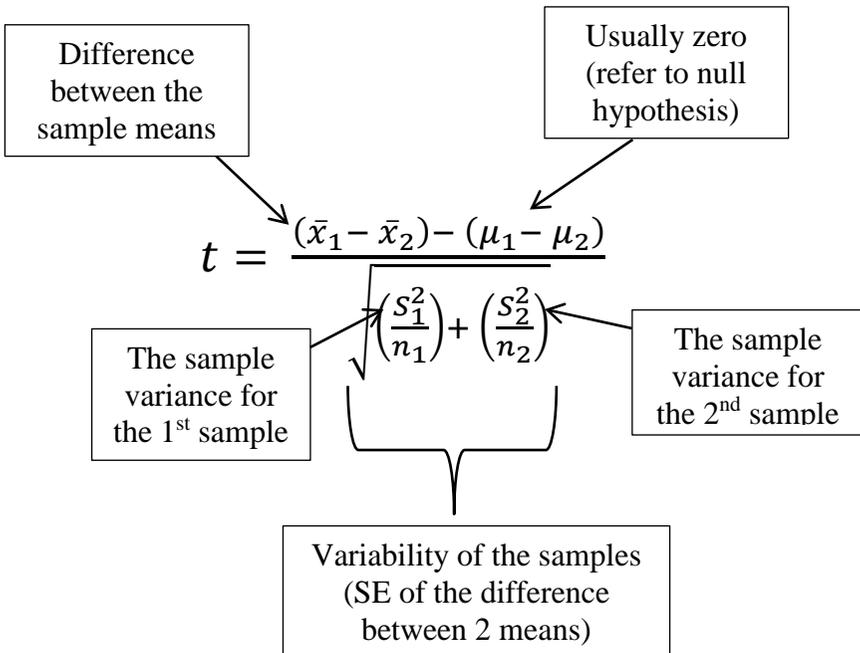
- The 2 samples (one from each population) are independent and random.
- Both populations are approximately normally distributed.
- The population variances are unknown but are about the same for both populations.

As with the one sample hypothesis test, we need to determine the critical  $t$ -value and then calculate a  $t$ -value to see if it lies in the reject  $H_0$  region.



The larger the difference between the means, the larger the calculated *t*-score, so the further out in the tail (past the critical *t*-score) the calculated *t*-score will lie.

The calculated *t*-score for a 2 sample *t*-test ( $\sigma$  unknown) is:



Thus

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\left(\frac{S_1^2}{n_1}\right) + \left(\frac{S_2^2}{n_2}\right)}}$$

**Example** Two stands are selected that were planted at the same time, one stand with a north facing aspect and one with a south facing aspect.

Stand 1

$$n_1 = 45 \text{ trees}$$

$$\bar{x}_1 = 37.7 \text{ ft}$$

$$s_1^2 = 1.58 \text{ ft}^2$$

Stand 2

$$n_2 = 45 \text{ trees}$$

$$\bar{x}_2 = 36.2 \text{ ft}$$

$$s_2^2 = 1.58 \text{ ft}^2$$

$$t = \frac{(37.7 - 36.2)}{\sqrt{\left(\frac{1.58}{45}\right) + \left(\frac{1.67}{45}\right)}}$$

$$t = \frac{1.5}{\sqrt{0.0351 + 0.0371}}$$

$$t = \frac{1.5}{\sqrt{0.0722}}$$

$$t = 5.5824$$

This is then compared to a  $t$ -value from a  $t$ -table.

$$t_c = t_{\alpha/2, (n_1 + n_2 - 2)}$$

$$t_c = t_{0.025,88} = 1.987$$

The hypothesis that the two means are the same is rejected if

$$|t| > |t_c|$$

Since  $5.5824 > 1.987$ , we reject the hypothesis and conclude that there is a significant difference in tree height between stands grown on north vs. south facing aspects.

## 10.10 HYPOTHESIS TESTING

Hypothesis testing is used to determine whether the hypothesis (assumption) made about a population is true or not. When we here that something is *statistically significant*, the statement is based on a hypothesis test.

*Null hypothesis ( $H_0$ )* – A hypothesis of no (null) difference. This is often the reverse of what we believe. It always contains an equality ( $=, \leq, \geq$ ).

*Alternative hypothesis ( $H_1$ )* – A hypothesis of what we want to prove. This never contains an equality ( $\neq, <, >$ ).

$\alpha$  – Acceptable probability of a Type I error, or the significance level (typically between 0.01 and 0.10). This determines the location of the rejection boundary.

*Type I error* – A false positive result. A rejection of the null hypothesis when, in fact, it is true.

$$H_0: \mu_P \geq \mu_{NP}$$

$$H_1: \mu_P < \mu_{NP}$$

For example, a Type I error might suggest that a pesticide reduces pest abundance when in fact it does not.

*Type II error* – A false negative result. The failure to reject a null hypothesis when in fact it is not true.

For example, a Type II error might suggest that a pesticide does not reduce pest abundance when in fact it does.

→ Increasing the sample size will reduce the potential for both types of error.

### Steps in hypothesis testing

1. Specify the null ( $H_0$ ) and alternative ( $H_1$ ) hypotheses.
2. Select a significance level ( $\alpha$ ).
3. Calculate the sample statistics associated with the population parameter expressed in the hypothesis.
4. Use the Student's  $t$  or the Z-distribution to assess the test.

### Examples of hypotheses:

“The white pine trees in Georgia have greater than 50% dieback”

$$H_0: \mu \leq 50\% \text{ Dieback}$$

$$H_1: \mu > 50\% \text{ Dieback}$$

“There is a difference in animal weights based on gender”

$$H_0: \mu_F = \mu_M$$

$$H_1: \mu_F \neq \mu_M$$

“Treating trees with pesticides reduces the abundance of pests”

$$H_0: \mu_P \geq \mu_{NP}$$

$$H_1: \mu_P < \mu_{NP}$$

### Notes about hypothesis testing

1. The null hypothesis will never be proven to be true, it can only be “failed to reject.” This is because we are already assuming that the null hypothesis is true and, rather than trying to prove our idea ( $H_1$ ) is correct, we must show that the null hypothesis is likely to be wrong. If you think about it, there is too much variability in natural resources for two separate populations to be equal or identical to each other.

Results from a hypothesis test will either be:

“Reject the null hypothesis”

or

“Fail to reject the null hypothesis”

2. If you increase the alpha ( $\alpha$ ) level (which you have control over), it becomes easier to reject the null hypothesis. But this means that you are accepting a higher chance for error in your conclusions. However, test results are stronger with a low  $\alpha$  (less than 0.10).
3. You may not need to know the formulas for hypothesis testing thanks to all of the available statistical software, but you do need to be able to recognize which analysis you should be running, which distribution you should use, etc. Questions you should ask yourself include:
  - Do I need to run a 1-sample, 2-sample, or paired  $t$ -test?
  - Do I need to look up a critical  $Z$ -score or  $t$ -score?
  - Is my test a one-tail or a two-tail test?
  - How do I interpret the results of hypothesis test?

## APPENDIX A ACRONYMS

To communicate effectively in natural resource management, one needs to become familiar with the common acronyms that are used in our working environment. Listed below are a number of the common acronyms used in this book.

ac	Acres
BA	Basal area
BAF	Basal area factor (prism sampling)
ch	Chains
CV	Coefficient of variation
cm	Centimeters
DBH	Diameter at breast height
d.i.b.	Diameter inside bark
d.o.b.	Diameter outside bark
EF	Expansion factor
ft	Feet
ft <sup>2</sup>	Square feet
ft <sup>3</sup>	Cubic feet
GIS	Geographic information system
GNSS	Global navigation satellite system
GPS	Global positioning system
ha	Hectares
H or ht	Tree height
km	Kilometers
in	Inches
m	Meters
m <sup>2</sup>	Square meters
m <sup>3</sup>	Cubic meters
Mh	Merchantable height
MBF	Thousand (M) board feet
PRF	Plot radius factor (prism sampling)
TF	Tree factor (prism sampling)
TPA	Trees per acre
TPH	Trees per hectare
VF	Volume factor (prism sampling)

**APPENDIX B CONVERSIONS AND EQUIVALENTS**

Conversions and equivalents are necessary in mapping, navigating, and assessing resource values. Communicating with others within our respective professions and from other countries can require conversion from English units to metric units.

Distances*English to English*

5,280 feet	=	1 mile
1,760 yards	=	1 mile
80 chains	=	1 mile
32 rods	=	1 mile
8,000 links	=	1 mile
100 links	=	1 chain
66 feet	=	1 chain
22 yards	=	1 chain

*Metric to English*

2.54 centimeters	=	1 inch
0.3048 meters	=	1 foot
20.11 meters	=	1 chain
1,609.3 meters	=	1 mile

*English to Metric*

0.3937 inches	=	1 centimeter
3.281 feet	=	1 meter
0.621 miles	=	1 kilometer

*Metric to Metric*

100 centimeters	=	1 meter
1,000 meters	=	1 kilometer

Areas*English to English*

10 chains <sup>2</sup>	=	1 acre
43,560 ft <sup>2</sup>	=	1 acre
640 acres	=	1 mile <sup>2</sup>
23,040 acres	=	1 standard township (PLSS)

*Metric to English*

0.0929 m <sup>2</sup>	=	1 ft <sup>2</sup>
0.4047 hectares	=	1 acre

*English to Metric*

2.471 acres	=	1 hectare
10.76 ft <sup>2</sup>	=	1 m <sup>2</sup>

*Metric to Metric*

10,000 m <sup>2</sup>	=	1 hectare
-----------------------	---	-----------

Volumes

35.32 ft <sup>3</sup>	=	1 m <sup>3</sup>
14.291 ft <sup>3</sup> / acre	=	1 m <sup>3</sup> / hectare
0.06997 m <sup>3</sup> / hectare	=	1 ft <sup>3</sup> / acre

Weights

2,000 pounds	=	1 ton
2,204.6 pounds	=	1 metric ton

## APPENDIX C WRITING A MEMO

This Appendix is designed to illustrate standard memo format and describe the numerous pitfalls students encounter when developing them.

### 1. STANDARD MEMO ELEMENTS

A standard memo contains five distinct elements:

1. The date that the memo was created.
2. The person who created the memo (you).
3. The person to whom the memo is addressed.
4. The subject of the memo.
5. The content of the memo (the most extensive part).

It is not imperative, but at the top of the memo, the word "Memorandum" can be placed.

When developing a memo from scratch (using a blank word processing form), you will need to type all of these elements yourself.

Microsoft Word provides a number of memo templates for you to use in developing memos related to class exercises. To access one of the templates, simply open Microsoft Word, then select **File**, then **New** from the menu system. In the "new document" dialog box, a "memo" tab should be available. Once you select this tab, you will be presented with several memo templates. Simply pick one by double-clicking it, and begin to develop your memo.

A sample (good) memo is provided on the next 3 pages.

## *Memorandum*

Date: 10/29/17

To: Pete Bettinger

From: Kenny Tougaw

Subject: Fixed-area plot sample of the Whitehall Seed Orchard

---

On Wednesday, October 24, 2017, David Chastain and I performed a fixed area plot sample of the Whitehall Seed Orchard. We designed a systematic sample with eleven 0.10-acre plots (Figure 1), and measured all of the live pine trees within those plots.

After analyzing the data, we estimate that the Seed Orchard contains 169 trees per acre with a basal area of 82 ft<sup>2</sup> per acre of live pine trees (Table 1). Our estimate of the volume is 1,975 ft<sup>3</sup> per acre.

Figure 1. Map of Whitehall Seed Orchard area sampled with fixed area plots

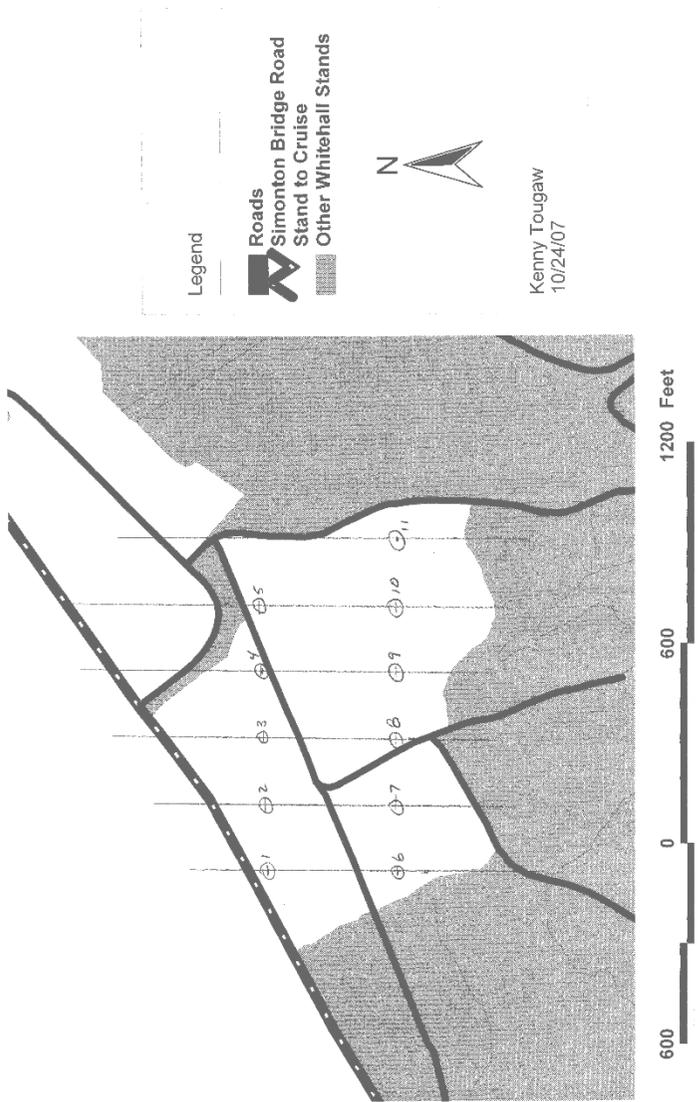


Table 1. Summary of fixed area sample of the Whitehall Seed Orchard

DBH (in)	Tally	TPA	Height (ft)	BA (ft <sup>2</sup> / ac)	Volume (ft <sup>3</sup> / ac)
6	14	12.7	37.2	2.5	43.9
7	23	20.9	41.9	5.6	110.2
8	32	29.1	45.8	10.2	218.7
9	35	31.8	49.1	14.1	324.4
10	31	28.2	51.9	15.4	374.9
11	25	22.7	54.4	15.0	382.8
12	19	17.3	56.4	13.6	359.6
13	7	6.4	58.3	5.9	160.7
Total	186	169.1		82.1	1975.1

### **1.A. THE DATE THAT THE MEMO WAS CREATED**

Make sure that the date the memo was created is correct. Common errors related to incorrectly dating the memo include:

- Using the incorrect year.
- Using a date for the memo that is prior to the date of data collection.
- Using the same date as previous memos (not updated when using a previous memo as a guide)

### **1.B. THE PERSON WHO CREATED THE MEMO (YOU)**

Make sure that you do not confuse this entry with the person to whom the memo should be addressed. Put your name here.

### **1.C. THE PERSON TO WHOM THE MEMO IS ADDRESSED**

Make sure that you place the appropriate name in the "To" entry. Do not use the following for FORS 3000 memos.

- "Instructor"
- "Lab TA"
- "Whom it may concern"

If you see an entry for "Cc:" in a memo template, this is where you would type the name of another person to which the memo will be copied and delivered.

### **1.D. THE SUBJECT OF THE MEMO**

Some memo templates use "Re:" instead of "Subject:" They are, in fact, the same thing. "Re" is simply shorthand for "regarding." Either of these is appropriate in a memo.

When writing the subject line, make it informative. Here are some good examples of a subject line:

- Prism sample of stand 294
- Strip sample for down wood in hardwood stand
- Compass and pacing results in four field conditions

Here are some poor examples of a subject line:

- Lab 3
- Lab results
- Seed Orchard
- Results
- October 24 lab

### **1.E. THE CONTENT OF THE MEMO**

The content of the memo should present information that conveys their results clearly, correctly, and concisely.

Please state in the content of the memo:

- a) The objective of the work ("David Chastain and I performed a fixed area plot sample of the Whitehall Seed Orchard").
- b) The approach used ("We designed a systematic sample with eleven 0.10-acre plots (Figure 1), and measured all of the live pine trees within those plots").
- c) A summary and conclusions ("After analyzing the data, we estimate that ....").

The following information is most often not appropriate in most memos for forestry and natural resource classes:

- The equipment you used, if it is standard equipment (diameter tape, clinometer, 100 foot tape). Non-standard equipment (laser range finder) could be mentioned.

- The steps you used to locate in the field your sample plots or points ("We walked 3 chains from the intersection of ....").

If figures or tables are to be attached to a memo,

- They must be referenced in the memo.
- They must be stapled to the memo.
- They must be placed in the order that they are referenced.
- They must have a title on them ("Figure 1. Map of the Whitehall Seed orchard fixed area plot sample").
- They must be relevant. A summary table is relevant. Ten intermediate tables (for individual plots, for example) that were used to create the summary table may not be relevant.

## **2. FINAL THOUGHTS**

To increase the professionalism of your memos, please take the following into consideration.

1. Use a standard font size.
2. Use a standard font (Times New Roman, Arial, etc.).
3. Spell-check your memo.
4. When using units such as ft<sup>2</sup>, make sure that the number has been superscripted (raised).
5. Provide the main results in the memo, leave the detail for a table.
6. Don't repeat, in a paragraph, all of the information that is included in a attached table. If a lot of detail is provided in the memo, consider bulleting the items.
7. Interpret the results.
8. Type the memo.
9. Single-space the paragraphs.
10. Do not indent the first line of the paragraphs.
11. Do not start the memo with "Dear Dr. Bettinger," or other pleasantries or salutations.

12. Do not end the memo with a pleasantry such as "Good bye" or "sincerely yours."
13. Do not use shorthand such as "ur" or "btw."
14. Use positive language where possible.
15. Capitalize "Internet", and products such as "Excel", and "Microsoft Word" when they are used in a memo.
16. Only provide what was requested. If what was requested was a memo and a graphs, do not attach anything else, such as Excel tables showing your work, to the memo.
17. Use commas in numbers over 999. For example, 123,897 is easier to read than 123897.

To increase the professionalism of your tables, please take the following into consideration.

1. Place column headers over the data that they represent.
2. Use a consistent precision within a column of data. Do not present values in a column of data with different decimal places.
3. Put the title of the table at the top of the table.
4. If attaching the table to a memo, your name and the date are not required on the table.
5. Use lines to separate the data from the column headers.
6. Use lines to close off the bottom of a table.
7. Spell-check the table.
8. Put the units of the data (inches, ft<sup>2</sup>, etc.) in the column header information.
9. Right-justify the data in a column - don't use a center- or left-justification.

To increase the professionalism of your figures, please take the following into consideration.

1. Label the axes of graphs.
2. Put the title of the figure at the top of the figure.
3. If attaching the figure to a memo, your name and the date are not required on the figure.

4. Use a font that is appropriate.
5. Spell-check the figure.
6. Put the units of the data (inches, ft<sup>2</sup>, etc.) in a graph with the axis label.

### **3. A POOR MEMO EXAMPLE**

On the next page is a poor example of a memo. Given the memo's brevity, and given the problems noted above, the student probably spent about 5 minutes on it. What is wrong?

1. The date does not contain the year.
2. The recipient's name is too informal ("Dr. B").
3. The subject is too vague ("Lab 10 results").
4. The content of the memo begins with an unnecessary salutation ("Hey").
5. The objectives are unclear - the stand sampled is unknown.
6. The approach is unclear (plot, strip, prism sample?).
7. The "lab partner" is unknown.
8. The first sentence is awkward.
9. The second sentence is lacking a period.
10. No actual results are provided in the content of the memo.
11. The table title is too brief.
12. One of the column headers is not spelled correctly.
13. No units (e.g., ft<sup>2</sup> / ac) for the columns are available.
14. Two of the column headers are not over the data they represent.
15. One column of data (BA) is center-justified, making the data hard to read.
16. One column of data (BA) has inconsistent precision - some data have one decimal place, some have no decimal places.
17. One column of data (Vol) is left-justified.
18. The trees per acre that were alluded to in the memo content are not found in the table.
19. The total basal area and volume per acre is not provided.
20. The paragraph is double-spaced and the first line is indented.

## *Memorandum*

Date: 10/29  
 To: Dr. B  
 From: John Smith  
 Subject: Lab 10 results

Hey,

After cruising the stand with my lab partner, I obtained trees per acre and volume for it. Table 1 shows these results

JS

Table 1. Results

DBH	Heighth	BA	Vol.
6	37	2.5	43.9
7	42	5.6	110.2
8	46	10.2	218.7
9	49	14.1	324.4
10	52	15.4	374.9
11	54	15	382.9
12	56	13.6	359.6
13	58	5.9	160.7

**ONCE AGAIN: This is a POOR example**

**APPENDIX D VOLUME AND WEIGHT EQUATIONS**I. Natural pine stands*a) Total green weight equation – Piedmont*

$$\text{Weight (pounds)} = 0.15179 (\text{DBH}^2)^{1.02917} H^{0.96160} \exp(-2.77201 d^{4.78156} \text{DBH}^{-5.01343})$$

Predicted weight of tree to a top diameter limit of  $d$  inches (o.b.).

*b) Sawlog green weight equation – Piedmont*

$$\text{Weight (pounds)} = 0.44802 (\text{DBH}^2)^{0.92193} Mh^{0.90894}$$

*c) Total cubic foot volume equation – Piedmont*

$$\text{Volume (ft}^3\text{)} = 0.00195 (\text{DBH}^2)^{1.00449} H^{1.02075} \exp(-3.0643 d^{4.65458} \text{DBH}^{-4.95963})$$

Predicted volume of tree to a top diameter limit of  $d$  inches (o.b.).

*d) Sawlog cubic foot volume equation – Piedmont*

$$\text{Volume (ft}^3\text{)} = 0.00676 (\text{DBH}^2)^{0.88791} Mh^{0.94669}$$

Variable definitions:

DBH = diameter at breast height (in)

H = total height (ft)

d = merchantable top diameter limit outside bark (o.b.) (in)

Mh = sawlog merchantable height (ft)

## II. Pine plantations

### *a) Loblolly pine green weight equation – Piedmont*

$$\text{Weight (pounds)} = 0.11576 (\text{DBH}^2\text{H})^{1.01935} \exp(-1.33918 d^{4.40499} \text{DBH}^{-4.24150})$$

Predicted green weight (lbs) of wood and bark to a top diameter limit of  $d$  inches (o.b.).

### *b) Loblolly Pine Cubic Volume Equation – Piedmont*

$$\text{Volume (ft}^3\text{)} = 0.00159(\text{DBH}^2\text{H})^{1.02725} \exp(-1.54900 d^{4.37037} \text{DBH}^{-4.29173})$$

Predicted cubic foot volume of wood and bark to a top diameter limit of  $d$  inches (o.b.).

Variable definitions:

DBH = diameter at breast height (in)

H = total height (ft)

$d$  = merchantable top diameter limit outside bark (o.b.) (in)

Pine equations come from Saucier et al. (1981), Clark and Saucier (1990), and Clark et al. (1991).

### III. Hardwoods

#### *a) Green weight*

Hardwood trees were classified as soft or hard hardwoods based on the green weight per cubic foot, branching habit, and specific gravity. Based on these criteria, red maple, sweetgum, sycamore, and yellow poplar were classified as soft hardwoods. Elm, hickory, chestnut oak, scarlet oak, southern red oak, and white oak were classified as hard hardwoods. The tables of coefficients that follow are those which we believe most would find useful. They use DBH and height to a 4 inch top to find weights and volumes to a 4 inch top, and DBH and sawlog merchantable height to find weights and volumes of the sawlog portion of the tree. Some representative tables which can be constructed from the coefficients and equation forms are presented.

Two equation forms are used to find volumes and weights of stem wood and bark, one for trees with  $DBH < 11$  inches and one for trees with  $DBH \geq 11$  inches.

For trees with  $DBH < 11$  inches:

$$Y = a'(DBH^2 H4)^c$$

Y = volume or weight

DBH = diameter at breast height (in)

H4 = tree height to 4 inch top (ft)

a' and c are coefficients (see table below)

For trees with  $DBH \geq 11$  inches:

$$Y = a''(DBH^2)^b H4^c$$

Y = volume or weight

DBH = diameter at breast height (in)

H4 = tree height to 4 inch top (ft)

a", b, and c are coefficients (see table below)

The coefficients listed in Table D-1 are from Table 16 of Clark et al. (1986) for estimating green weight of total stem wood and bark for hardwood species in the Piedmont.

Table D-1. Green weight of total stem wood and bark for hardwood tree species in the Piedmont

Species	a'	c	a"	b
Soft hardwoods	2.29156	0.70545	0.42879	1.05493
Red maple	2.22532	0.69908	0.53744	0.99535
Sweetgum	2.50069	0.69517	0.36891	1.09422
Sycamore	1.57648	0.75799	0.59202	0.96221
Yellow-poplar	1.82487	0.73310	0.47036	0.47036
Hard hardwoods	2.16281	0.72694	0.41701	1.07017
Elm species	2.12287	0.71366		
Hickory species	2.69454	0.69078	0.20666	1.22623
Chestnut oak	1.86199	0.73296		
Scarlet oak	2.59832	0.71047	0.45824	1.07229
South. red oak	2.89546	0.69112	0.62603	1.01046
White oak	1.85973	0.74792	0.43602	1.05037
All hardwoods	2.30521	0.71114	0.42329	1.06454

**Example** What is the weight of a 10 inch DBH sweetgum with a 4" top height of 60 feet?

$$Y = 2.50069 (10^2 60)^{0.69517}$$

$$Y = 1,058 \text{ pounds}$$

**Example** What is the weight of a 10 inch DBH white oak with a 4" top height of 60 feet?

$$Y = 1.85973 (10^2 60)^{0.74792}$$

$$Y = 1,245 \text{ pounds}$$

**Example** What is the weight of a 12 inch DBH sweetgum with a 4" top height of 60 feet?

$$Y = 0.36891 (12^2)^{1.09422} 60^{0.69517}$$

$$Y = 1,461 \text{ pounds}$$

These previous equations give the weight of the whole tree stem. To find the weight to any top diameter limit, the weight of the whole tree stem must be multiplied by a ratio, found by the following equation:

$$Y_R = \exp(a (d^b) DBH^c)$$

$Y_R$  = the ratio of volume or weight to a top diameter of  $d$  inches to the whole tree stem volume or weight

DBH = diameter at breast height (in)

$a$ ,  $b$ , and  $c$  are coefficients.

The coefficients listed in Table D-2 are from Clark et al. (1986) for estimating green weight of total stem wood and bark for hardwood species in the Piedmont.

Table D-2. Green weight coefficients for the merchantable weight ratio equations

Species	a	b	c
Soft hardwoods	-1.73265	4.21073	-4.28793
Red maple	-0.80857	4.37677	-4.10402
Sweetgum	-1.88706	4.09280	-4.20271
Sycamore	-2.31104	4.80579	-4.88417
Yellow-poplar	-1.72938	3.99726	-4.17198
Hard hardwoods	-2.61988	3.92928	-4.33719
Elm species	-5.22488	4.41024	-4.91101
Hickory species	-9.21973	4.07639	-5.02575
Chestnut oak	-1.26318	4.69218	-4.69208
Scarlet oak	-4.43019	3.68458	-4.45247
South. red oak	-2.08555	3.49897	-3.80231
White oak	-1.99440	3.91077	-4.20029
All hardwoods	-2.25900	4.00280	-4.27579

**Example** For a sweetgum with DBH = 10 inches, what proportion of the total weight is in the tree stem below a 4 inch top, and how much does that weigh?

$$Y_R = \exp(-1.88706 (4^{4.09280}) 10^{-4.20271})$$

$$Y_R = 0.9661$$

So, the green weight to a 4 inch top is just the total stem green weight multiplied by the proportion of the total weight up to a 4 inch top. For the 10 inch sweetgum with a 4 inch top height of 60 feet we earlier calculated the total stem weight as 1,058 lbs. Therefore, the estimated green weight to a 4 inch top is:

$$\text{Weight} = 1,058 \text{ pounds} (0.9661) = 1,022 \text{ pounds}$$

*b) Cubic foot volume*

The equation forms for cubic foot volume are the same as for green weight, with one equation form for trees with DBH < 11 inches and a different equation form for trees with DBH  $\geq$  11 inches. The coefficients for cubic foot volume estimation, as presented in Table 18 of Clark et al. (1986) are listed in Table D-3.

Table D-3. Cubic foot volume of total stem wood and bark for hardwood tree species in the Piedmont

Species	a'	c	a''	b
Soft hardwoods	0.03907	0.70612	0.00887	1.01517
Red maple	0.03279	0.72602	0.01353	0.91057
Sweetgum	0.04392	0.68974	0.00827	1.03783
Sycamore	0.02603	0.75115	0.01091	0.93239
Yellow-poplar	0.03569	0.72218	0.00917	1.00549
Hard hardwoods	0.03434	0.72578	0.00825	1.02317
Elm species	0.03122	0.73147		
Hickory species	0.05130	0.67784	0.00580	1.13323
Chestnut oak	0.02915	0.74106		
Scarlet oak	0.03696	0.71620	0.00649	1.07908
South. red oak	0.04688	0.68265	0.00832	1.04325
White oak	0.03008	0.74210	0.00848	1.00605
All hardwoods	0.03725	0.71359	0.00861	1.01890

These coefficients, when used in the equation forms presented in the section on green weights, produce estimates of total stem wood and bark cubic foot volume.

To estimate the cubic foot volume to any top diameter limit, the same ratio equation form is used as described above for green weight equations. The coefficients, as presented in Table 24 of Clark et al. (1986) are listed in Table D-4.

Table D-4. Cubic foot volume coefficients for the merchantable volume ratio equations

Species	a	b	c
Soft hardwoods	-1.65202	4.16510	-4.23243
Red maple	-0.63491	4.07791	-3.74763
Sweetgum	-1.71771	4.08636	-4.16133
Sycamore	-2.51196	4.72070	-4.84581
Yellow-poplar	-1.76181	3.97649	-4.16652
Hard hardwoods	-2.69221	3.89875	-4.32412
Elm species	-5.53946	4.23262	-4.74180
Hickory species	-9.21460	4.04604	-4.99672
Chestnut oak	-1.38721	4.89309	-4.91657
Scarlet oak	-4.61107	3.52799	-4.33254
South. red oak	-2.20401	3.43172	-3.77125
White oak	-2.17082	3.89490	-4.21939
All hardwoods	-2.21693	3.94846	-4.22458

#### IV. Product-related conversion factors

Sources for conversions used here are Williams and Hopkins (1968) and Wenger (1984).

##### *a) Pine sawtimber and large logs*

15,000 lbs (range 13,000-17,000 lbs) per MBF Scribner rule  
16,000 lbs per MBF Doyle rule  
12,450 lbs per MBF International 1/4 inch rule

##### *b) Pine chip-n-saw*

15,000 lbs (range 13,000-17,000 lbs) per MBF Scribner rule  
19,950 lbs per MBF Doyle  
12,450 lbs per MBF International 1/4 inch rule

##### *c) Pine pulpwood and chip-n-saw*

5,350 lbs (range 5,000-5,620 lbs) per standard cord  
2.80 cords per MBF (Scribner rule)

##### *d) Hardwood sawtimber*

17,500 lbs (range 15,000-19,000 lbs) per MBF Doyle rule  
13,125 lbs per MBF Scribner rule  
10,850 lbs per MBF International 1/4 inch rule

##### *e) Hardwood pulpwood*

5,800 lbs (range 5,400-6,075 lbs) per standard cord  
3.02 cords per MBF (Doyle rule)

##### *f) English and metric conversions*

1 short ton (2,000 lbs) of green southern pine, wood and bark, has about 0.822 m<sup>3</sup> of solid wood.

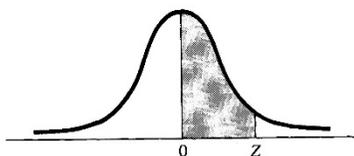
1 short ton (2,000 lbs) of green mixed hardwood, wood and bark,  
has about 0.787 m<sup>3</sup> of solid wood.  
1 metric tonne = 1.102 short tons = 2,204 pounds

## REFERENCES

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## APPENDIX E STANDARD NORMAL (Z) AND STUDENT'S T DISTRIBUTION TABLES

TABLE E.2. The standardized normal distribution

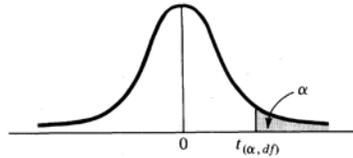


Entry represents area under the standardized normal distribution from the mean to Z

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2267	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.49865	.49869	.49874	.49878	.49882	.49886	.49889	.49893	.49897	.49900
3.1	.49903	.49906	.49910	.49913	.49916	.49918	.49921	.49924	.49926	.49929
3.2	.49931	.49934	.49936	.49938	.49940	.49942	.49944	.49946	.49948	.49950
3.3	.49952	.49953	.49955	.49957	.49958	.49960	.49961	.49962	.49964	.49965
3.4	.49966	.49968	.49969	.49970	.49971	.49972	.49973	.49974	.49975	.49976
3.5	.49977	.49978	.49978	.49979	.49980	.49981	.49981	.49982	.49983	.49983
3.6	.49984	.49985	.49985	.49986	.49986	.49987	.49987	.49988	.49988	.49989
3.7	.49989	.49990	.49990	.49990	.49991	.49991	.49992	.49992	.49992	.49992
3.8	.49993	.49993	.49993	.49994	.49994	.49994	.49994	.49995	.49995	.49995
3.9	.49995	.49995	.49996	.49996	.49996	.49996	.49996	.49996	.49997	.49997

**TABLE E.3** Critical values of  $t$

For a particular number of degrees of freedom, entry represents the critical value of  $t$  corresponding to a specified upper tail area  $\alpha$



Degrees of Freedom	Upper Tail Areas					
	.25	.10	.05	.025	.01	.005
1	1.0000	3.0777	6.3138	12.7062	31.8207	63.6574
2	0.8165	1.8856	2.9200	4.3027	6.9646	9.9248
3	0.7649	1.6377	2.3534	3.1824	4.5407	5.8409
4	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
5	0.7267	1.4759	2.0150	2.5706	3.3649	4.0322
6	0.7176	1.4398	1.9432	2.4469	3.1427	3.7074
7	0.7111	1.4149	1.8946	2.3646	2.9980	3.4995
8	0.7064	1.3968	1.8595	2.3060	2.8965	3.3554
9	0.7027	1.3830	1.8331	2.2622	2.8214	3.2498
10	0.6998	1.3722	1.8125	2.2281	2.7638	3.1693
11	0.6974	1.3634	1.7959	2.2010	2.7181	3.1058
12	0.6955	1.3562	1.7823	2.1788	2.6810	3.0545
13	0.6938	1.3502	1.7709	2.1604	2.6503	3.0123
14	0.6924	1.3450	1.7613	2.1448	2.6245	2.9768
15	0.6912	1.3406	1.7531	2.1315	2.6025	2.9467
16	0.6901	1.3368	1.7459	2.1199	2.5835	2.9208
17	0.6892	1.3334	1.7396	2.1098	2.5669	2.8982
18	0.6884	1.3304	1.7341	2.1009	2.5524	2.8784
19	0.6876	1.3277	1.7291	2.0930	2.5395	2.8609
20	0.6870	1.3253	1.7247	2.0860	2.5280	2.8453
21	0.6864	1.3232	1.7207	2.0796	2.5177	2.8314
22	0.6858	1.3212	1.7171	2.0739	2.5083	2.8188
23	0.6853	1.3195	1.7139	2.0687	2.4999	2.8073
24	0.6848	1.3178	1.7109	2.0639	2.4922	2.7969
25	0.6844	1.3163	1.7081	2.0595	2.4851	2.7874
26	0.6840	1.3150	1.7056	2.0555	2.4786	2.7787
27	0.6837	1.3137	1.7033	2.0518	2.4727	2.7707
28	0.6834	1.3125	1.7011	2.0484	2.4671	2.7633
29	0.6830	1.3114	1.6991	2.0452	2.4620	2.7564
30	0.6828	1.3104	1.6973	2.0423	2.4573	2.7500
31	0.6825	1.3095	1.6955	2.0395	2.4528	2.7440
32	0.6822	1.3086	1.6939	2.0369	2.4487	2.7385
33	0.6820	1.3077	1.6924	2.0345	2.4448	2.7333
34	0.6818	1.3070	1.6909	2.0322	2.4411	2.7284
35	0.6816	1.3062	1.6896	2.0301	2.4377	2.7238
36	0.6814	1.3055	1.6883	2.0281	2.4345	2.7195
37	0.6812	1.3049	1.6871	2.0262	2.4314	2.7154
38	0.6810	1.3042	1.6860	2.0244	2.4286	2.7116
39	0.6808	1.3036	1.6849	2.0227	2.4258	2.7079
40	0.6807	1.3031	1.6839	2.0211	2.4233	2.7045
41	0.6805	1.3025	1.6829	2.0195	2.4208	2.7012
42	0.6804	1.3020	1.6820	2.0181	2.4185	2.6981
43	0.6802	1.3016	1.6811	2.0167	2.4163	2.6951
44	0.6801	1.3011	1.6802	2.0154	2.4141	2.6923
45	0.6800	1.3006	1.6794	2.0141	2.4121	2.6896

For 95% confidence

**APPENDIX F STATISTICS SYMBOLS AND FORMULAS**Symbols for populations

$\mu$ (mu)	Population mean
$\sigma$ (sigma)	Population standard deviation
$\sigma^2$	Population variance
$N$	Total number in a population

Symbols for samples

$n$	Total number in a sample
$\bar{x}$	Sample mean
$s$	Sample standard deviation ( <i>SD</i> )
$s^2$	Sample variance
$s_{\bar{x}}$	Sample standard error ( <i>SE</i> )
$\nu$ (nu)	Degrees of freedom ( <i>df</i> )
$\alpha$ (alpha)	Significance level in hypothesis testing or acceptable probability of error

Formulas for population parameters

Population mean:  $\mu = \frac{\sum x_i}{N}$

Population variance:  $\sigma^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{N}}{N}$  or  $\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$

Population standard deviation (SD):  $\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum x_i^2 - \frac{(\sum x_i)^2}{N}}{N}}$

Population coefficient of variation:  $CV = \left(\frac{\sigma}{\mu}\right) * 100\%$

### Formulas for sample estimates

Sample mean:  $\bar{x} = \frac{\sum x_i}{n}$

Sample variance:  $s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$  or  $s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$

Sample standard deviation:  $s = \sqrt{s^2} = \sqrt{\frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}}$

Sample coefficient of variation:  $CV = \left(\frac{s}{\bar{x}}\right) * 100\%$

Standard error of the mean:  $s_{\bar{x}} = \frac{s}{\sqrt{n}}$

### Formulas related to statistical distributions

Z-score – population parameters known:  $Z = \frac{x - \mu}{\sigma}$

Z-score – population parameters unknown:  $Z = \frac{x - \bar{x}}{s}$

2-tail critical Z-score:  $Z_c = Z_{(1-\alpha)/2}$

1-tail critical Z-score:  $Z_c = Z_{0.5-\alpha}$

2-tail critical t-score, 1-sample:  $t_c = t_{\alpha/2, n-1}$

1-tail critical t-score, 1-sample:  $t_c = t_{\alpha, n-1}$

1-sample calculated Z-score:  $Z = \frac{\bar{x} - \mu_{H_0}}{SE}$

1-sample calculated t-score:  $t = \frac{\bar{x} - \mu_{H_0}}{SE}$

$$\text{2-sample calculated } t\text{-score: } t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}}$$

$$\text{2-tail critical } t\text{-score, 2-sample: } t_c = t_{\alpha/2, (n_1 + n_2 - 2)}$$

$$\text{1-tail critical } t\text{-score, 2-sample: } t_c = t_{\alpha, (n_1 + n_2 - 2)}$$

$$\text{Paired 1-sample calculated } t\text{-score: } t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

$$\text{Paired 2-sample calculated } t\text{-score: } t = \frac{(\bar{d}_1 - \bar{d}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{s_{d1}^2}{n_1}\right) + \left(\frac{s_{d2}^2}{n_2}\right)}}$$

### Formulas for confidence intervals

Confidence interval – population parameters known or sample size is greater than 30:  $CI = \bar{x} \pm Z_{(1-\alpha)/2}(s_{\bar{x}})$

Confidence interval – population parameters unknown and sample size is less than 30:  $CI = \bar{x} \pm t_{n-1, (\alpha/2)}(s_{\bar{x}})$

### Formula for tests of differences between sample means

One-interval method for comparison of means:

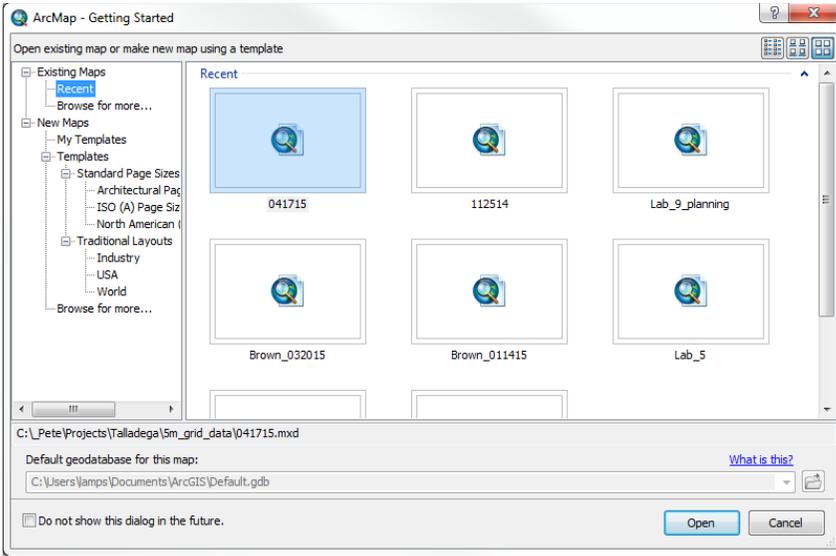
$$CI_{diff} = (\bar{x}_1 - \bar{x}_2) \pm Z_{(1-\alpha)/2} \left( \sqrt{(s_{\bar{x}_1})^2 + (s_{\bar{x}_2})^2} \right)$$

If the interval includes zero, the means are *not* significantly different.

## APPENDIX G MAKING A MAP USING ARCMAP

The following section is provided to enable you to make a map using ArcMap. We assume here that the field data have been saved as *shapefiles*.

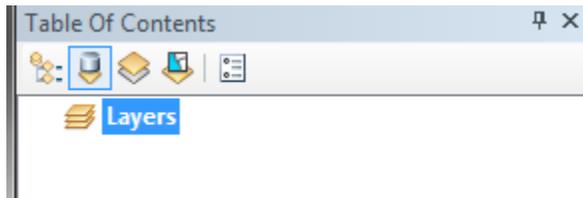
### 1. Open ArcMap



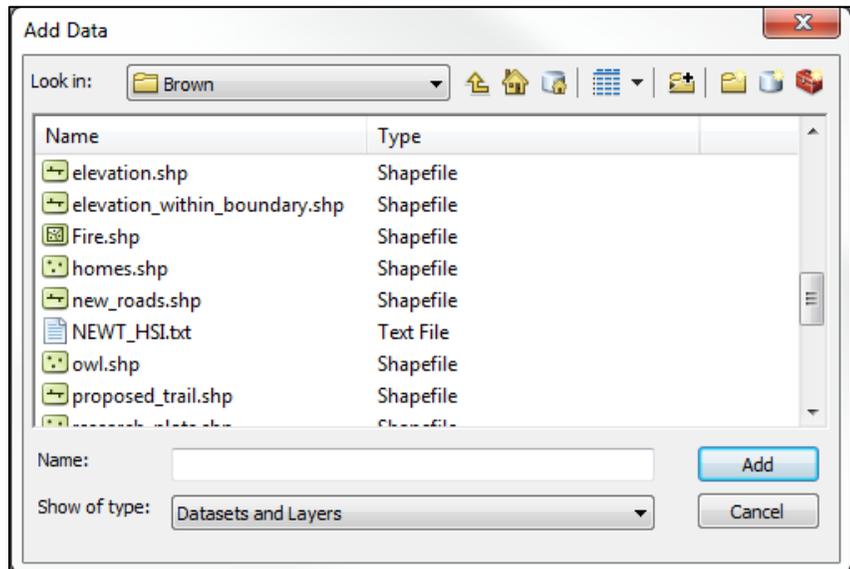
All of the following instructions refer to the ArcMap module of ArcGIS. To begin a new map, cancel the *Getting Started* dialog box. If a map (project) has already been saved, select *An Existing Map*. A "map" is simply a file that remembers the databases that are open, and how they are displayed.

## 2. Add a database to the View

Right-click the "Layers" noted in the table of contents,



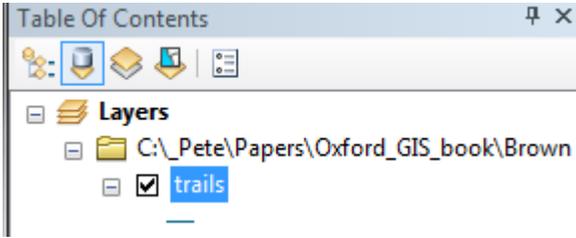
and select *Add Data*. An Add Data dialog box should appear. Search your computer or network storage space to find the appropriate shapefile that you want to use to create a map.



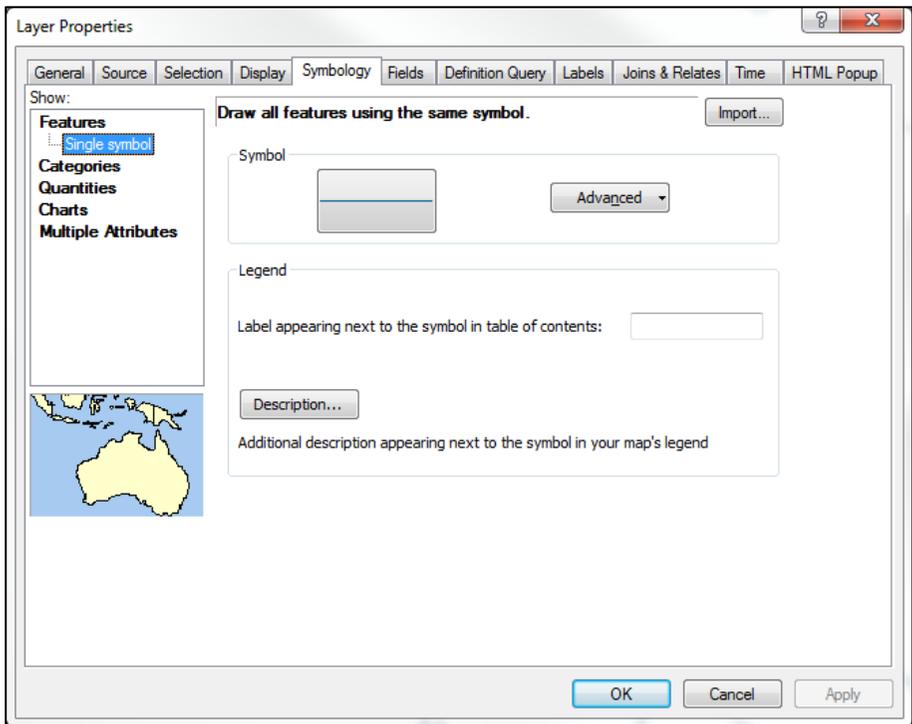
If a message appears indicating that the data is not spatially referenced, you will need to define the coordinate and projection system (Step 4).

3. Change the display characteristics of the data contained in the shapefile.

Right-click the shapefile that is listed under "Layers" in the Table of Contents.

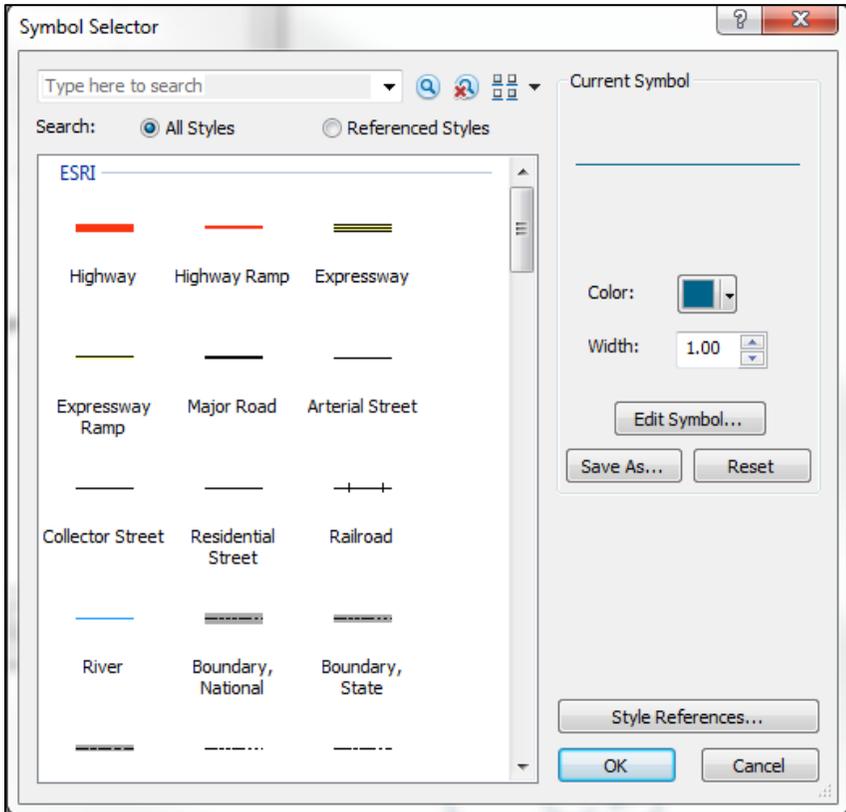


Select *Properties*. A Layer Properties dialog box should appear.



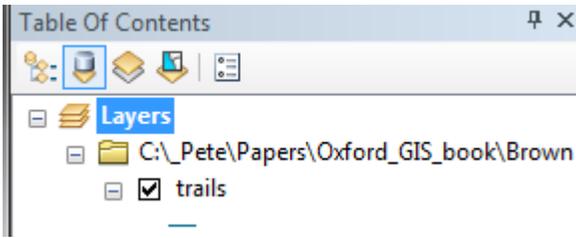
After selecting the *Symbology Tab*, click the *Symbol* command

button. The Symbol Selector dialog box should appear. Options are now available to change the thickness (width) and color of the features contained in the shapefile. Alternatively, you can choose from one of the pre-defined symbol types. When completed, press *OK*, then *Apply* in the Layer Properties dialog box, then exit the dialog box.

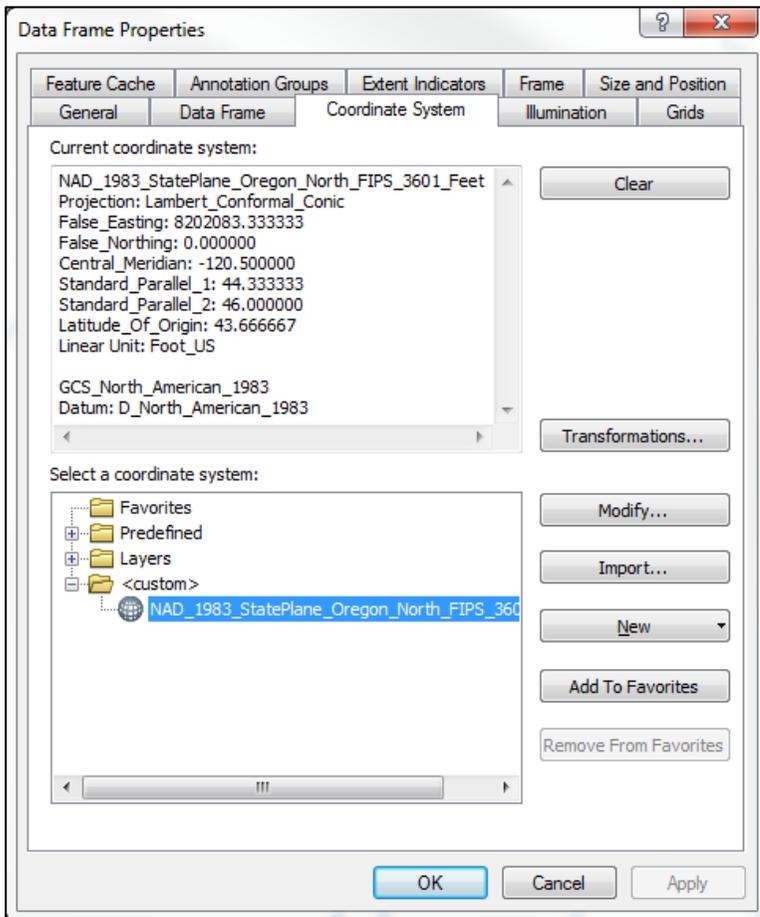


#### 4. Change the coordinate system

If the coordinate and projection systems need to be defined, right-click the "Layers" in the Table of Contents, and select *Properties*.



A Data Frame Properties dialog box should appear. Here, select the Coordinate System tab, and navigate through the predefined coordinate systems to the one that represents the data being displayed.



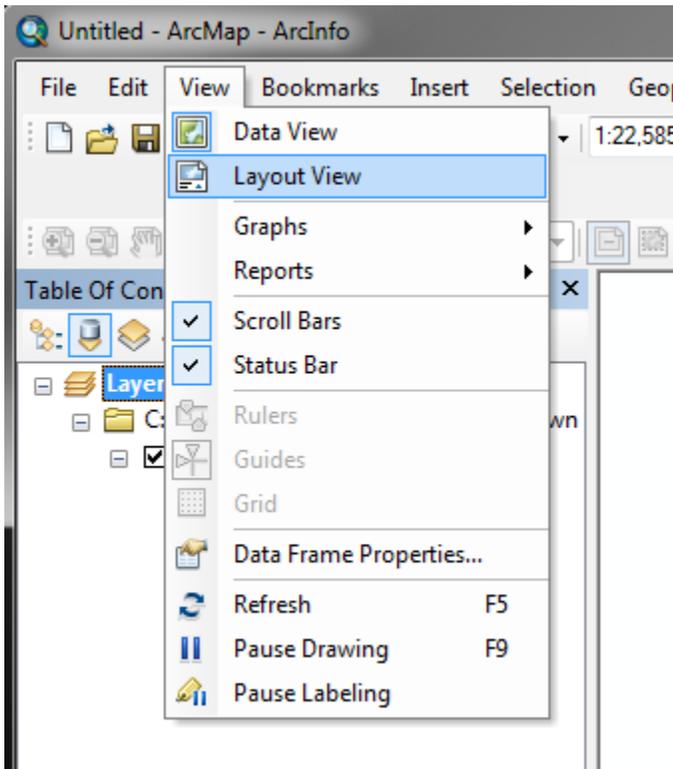
### 5. Zoom in to the area of interest (if necessary).

Use the *Zoom In* and *Zoom Out* and *Pan* buttons to position in the center of the Data View window the features that you want to be contained in map.

The *Zoom in*, *Zoom Out*, and *Pan* buttons can be found below the Main Menu system, on the left-hand side of the first row of buttons.

### 6. Make a map.

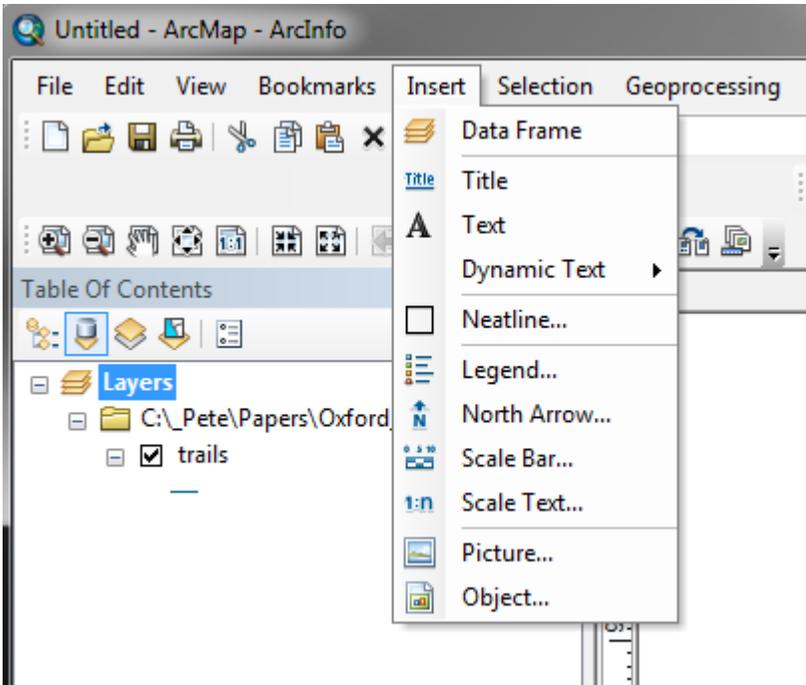
Select *View*, then *Layout View* from the Menu System.



Rather than working in the Data View window, we are now working in the Layout View window. What you see in the Layout Window is what the map will look like.

To change the orientation of the map (e.g., from portrait to landscape), right-click somewhere just outside the printable page and select Change Layout. You will be presented with a number of options to change the map's orientation and appearance.

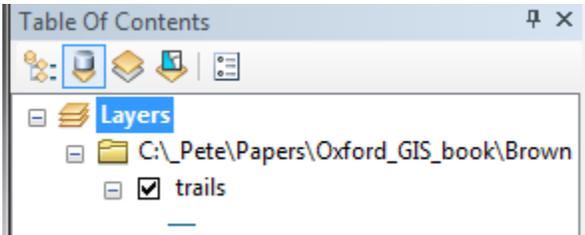
The map may not initially contain a scale, a north arrow, nor a legend. To insert these objects, select *Insert* from the Main Menu system, and insert each of these objects.



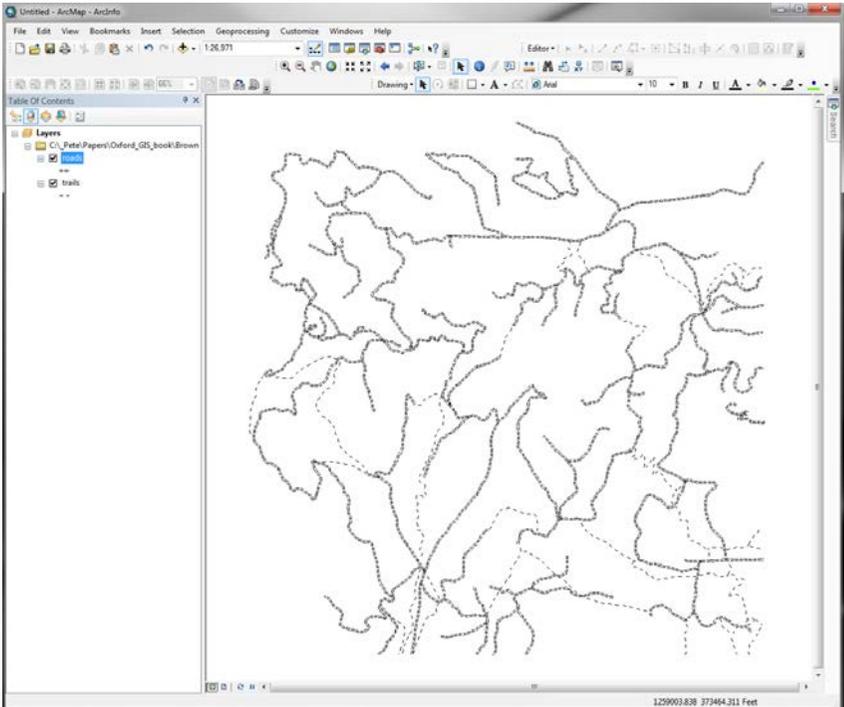
Each of these items can be re-sized by single-clicking them, as can the properties of each. Each of these items can also be moved by single-clicking them, grabbing them and moving them.

7. Add other shapefiles to the project.

If other shapefiles should be added to the map, you need to return to the Data View and add them. To do so, select *View*, then select *Data View*. Then right-click the "Layers" in the Table of Contents, and select *Add Data*.



The Add Data theme dialog box appears. The added shapefiles will likely need their display characteristics changed (step 3).



To return to the Layout Window, where the map is being made, select *View*, then select *Layout View*.

After adding new databases, some of the objects on the map (scale, north arrow, etc.) may need to be moved to other places on the map.

### 8. Important points.

1. The font size of the objects in the Layout Window needs to be reasonable. Print out a map and check the appearance on paper. If the fonts sizes seem too big or too small, adjust them.
2. The arrangement of items on the map should not be haphazard. Move them around to make the map look better.
3. The colors and characteristics of the features in the map should be reasonably chosen so that each feature is easy to see by a customer of the map. If they are not easily seen (e.g., the colors are so similar that it is hard to separate roads from trails), then change them (step 3). The colors you see on the computer screen may be printed differently.
4. Annotation can (should) be added to the map. Your name, the date, and other pertinent information can be added to the map in the Layout Window by selecting the *Insert* from the Menu System, then *Text*, then click an area of the map where the text may reside and type the text.
5. Other types of databases can be added to the Data View, including ArcInfo coverages, digital orthophotographs, and others.
6. Save the map often. The "map file" contains all of the information regarding how the shapefiles are presented in the Data View, and how the map is arranged in the Layout View.

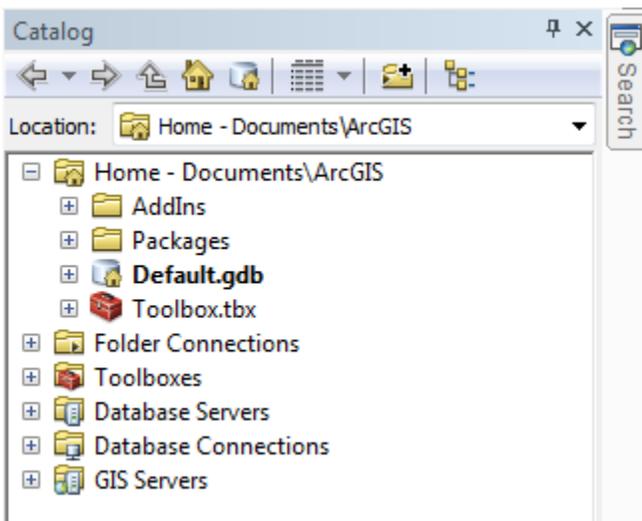
## 9. Develop a new theme of landscape features

To create a new shapefile from scratch in ArcMap, first select the ArcCatalog button from the first row of buttons below the Menu System.

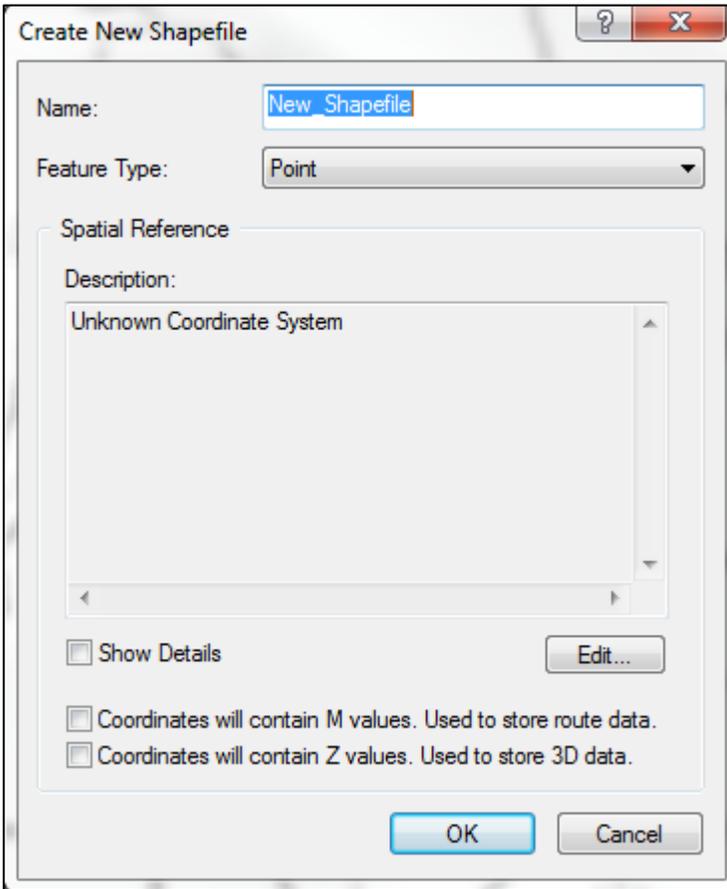


You will be taken to ArcCatalog, another module within the ArcGIS system, which is separate from ArcMap.

From within ArcCatalog, first navigate in the left-hand window to the directory / subdirectory where you want the new shapefile to be stored.



Then, from the Menu System, select *File*, then *Select New*, then *Shapefile*. A Create New Shapefile dialog box should appear.



Here, you type in the name of the new shapefile, and select the type of features that will be stored in the shapefile. After you press *OK*, an empty shapefile will have been created using the name you provided.

At this point, you can return to ArcMap and begin adding features to the empty shapefile. While we have demonstrated how to do this earlier, here is a short list of instructions:

1. Right-click Layers in the Table of Contents, and Add Data.
2. Add the new, empty shapefile to the Data View.
3. Select *Editor*, then *Start Editing*.

4. Select a task, such as *Create New Feature*.
5. Set the snapping options and parameters.
6. Draw new features.
7. Save edits.
8. Stop editing when you are done.

In ArcMap, you can snap features of one shapefile to other features in a separate shapefile.